ALTAM Fall 2023 Model Solutions

ALTAM Fall 2023 Q1

(a)
$${}_{20}E_{65:65}=\left({}_{20}E_{65}\right)^2(1.05)^{20}=(0.24381)^2(1.05)^{20}=0.157727$$

or

$${}_{20}E_{65:65}=\left({}_{20}p_{65}\right)^2(1.05)^{-20}=0.157727$$

$${}_{20}p_{65}=\frac{61{,}184.9}{94{,}579.7}=0.64691$$

(b) $A_{65:65:\overline{20|}}=A_{65:65}-{}_{20}E_{65:65}\,A_{85:85}+{}_{20}E_{65:65}=0.480488$

$$\ddot{a}_{65:65:\overline{20|}}=\ddot{a}_{65:65}-{}_{20}E_{65:65}\ddot{a}_{85:85}=10.9098$$

or $\quad \ddot{a}_{65:65:\overline{20|}}=\dfrac{1-A_{65:65:\overline{20|}}}{d}=10.9098$

$$P=\frac{100{,}000(0.480488)+500}{0.95(10.9098)-0.15}=4753.04$$

$A_{65:65}=0.44366 \qquad A_{85:85}=0.77652 \qquad \ddot{a}_{65:65}=11.6831 \qquad \ddot{a}_{85:85}=4.903$

(c) $A_{75:75:\overline{10|}}=1-d\ddot{a}_{75:75:\overline{10|}}=0.683032$

$${}_{10}V=100{,}000A_{75:75:\overline{10|}}-0.95P\ddot{a}_{75:75:\overline{10|}}=38{,}247.28$$

$\ddot{a}_{75:75}=6.6563 \qquad d=0.0476$

(d) The gain is

$$100\big(38{,}247.28 + 0.95(4{,}753.04)\big)(1.055) - 4(100{,}100) - 96(42{,}811.92)$$

$$= 1{,}117.12$$

(e)

The probability that at least one of the lives dies in the 11th year is

$$q_{75:75} = 1 - p_{75:75} = 1 - (0.981567)^2 = 0.036526$$

The interest gain is Profit using actual interest, expected mortality and expected expenses – Profit using expected interest, mortality and expenses, i.e.

$$100\big(38{,}247.28 + 0.95(4{,}753.04)\big)(1.055) - 100q_{75:75}(100{,}000) - 100p_{75:75}(42{,}811.92)$$

$$-\Big(100\big(38{,}247.28 + 0.95(4{,}753.04)\big)(1.05) - 100q_{75:75}(100{,}000) - 100p_{75:75}(42{,}811.92)\Big)$$

$$= 100(38{,}247.28 + 0.95(4{,}753.04))(0.005) = 21{,}381.33$$

The mortality gain is Profit using actual interest, actual mortality and expected expenses – Profit using actual interest, expected mortality and expected expenses, i.e.

$$100\big(38{,}247.28 + 0.95(4{,}753.04)\big)(1.055) - 4(100{,}000) - 96(42{,}811.92)$$

$$-\Big(100\big(38{,}247.28 + 0.95(4{,}753.04)\big)(1.055) - 100q_{75:75}(100{,}000) - 100p_{75:75}(42{,}811.92)\Big)$$

$$= (100q_{75:75} - 4)(100{,}000 - 42{,}811.92) = -19{,}864.11$$

The expense gain is Profit using actual interest, actual mortality and actual expenses – Profit using actual interest, actual mortality and expected expenses, i.e.

$$100\big(38{,}247.28 + 0.95(4{,}753.04)\big)(1.055) - 4(100{,}100) - 96(42{,}811.92)$$

$$-\Big(100\big(38{,}247.28 + 0.95(4{,}753.04)\big)(1.055) - 4(100{,}000) - 96(42{,}811.92)\Big)$$

$$= -\,400.00$$

So we have an interest gain of 21,381, a mortality loss of 19,864 and an expense loss of 400, giving a total of $21,381 - 19,864 - 400 = 1,117$ as required.

(f) (i) $\text{Profit}_{20} = 40\big((19V + 0.95P)(1.05)\big) - 35(100{,}000) - 5(100{,}000)$

$$= 40\big((19V + 0.95P)(1.05) - 100{,}000\big)$$

$$_{19}V = 100{,}000v - 0.95P$$

$$\Rightarrow \ \text{Profit}_{20} = 40(100{,}000v(1.05) - 100{,}000) = 0$$

(ii) A major pandemic would have no impact on the mortality gain/loss in the final year, since the benefit of 100,000 is paid at the year end regardless of the survival or death of the policyholders (as endowment if both partners survive, as death claim if they do not).

Examiners' Comments

Overall, this question was done very well, especially for parts (a), (b), and (c). For parts (b) and (c), the most common mistakes were related to incorrectly handling expenses and miscalculating the endowment EPV. Many candidates did confuse the notation for the endowment EPV with that of a term insurance, though this was not penalized if the calculations were done correctly.

Parts (d) and (e) were generally done well, with common mistakes including incorrectly calculating the mortality rate, and calculating the gain / loss for a single policy rather than for the portfolio.

For part (f), some candidates calculated the total gain directly, while others calculated the gain by source and then combined them; both approaches were correct. Most candidates were able to identify that the endowment policy being in its final year resulted in no mortality loss.

ALTAM Fall 2023 Q2

(a)

For Option A,

- Model 2 could be used because it allows for a payment upon CI diagnosis, and then two different payment amounts on death, depending on whether the CI benefit had previously been paid.
- Model 1 doesn't work for this option because if the CI benefit is paid, the model doesn't allow for a subsequent DB payment;
- Model 3 doesn't work because it does not distinguish between deaths after CI or deaths without a CI diagnosis, which matters for this option.

For Option B,

- Model 1 or Model 2 or Model 3 could work.

For Option C,

- Either Model 2 or Model 3 could work.
- Model 1 would not work for this option because it only allows for a single payment, and two payments are necessary in the event that the insured receives a CI diagnosis prior to death.

(b) (i)

$$\int_0^{10} \left({}_tp_{50}^{00}\,\mu_{50+t}^{02} + {}_tp_{50}^{01}\,\mu_{50+t}^{12} \right) dt$$

$$\int_0^{10} {}_tp_{50}^{00} \left(\mu_{50+t}^{02} + \mu_{50+t}^{01}\ {}_{10-t}p_{50+t}^{12} \right) dt$$

$$\int_0^{10} {}_tp_{50}^{00} \left(\mu_{50+t}^{02} + \mu_{50+t}^{01} \int_0^{10-t} {}_rp_{50+t}^{11}\,\mu_{50+t+r}^{12}\,dr \right) dt$$

(ii)

$$\int_0^{10} 100,000 \left({}_tp_{50}^{00}\,\mu_{50+t}^{02} + {}_tp_{50}^{01}\,\mu_{50+t}^{12} \right) v^t dt$$

(c)

$$\ddot{a}_{50}^{00} \approx \bar{a}_{50}^{00} + \frac{1}{2} + \frac{\mu_{50}^{0\bullet} + \delta}{12} = 13.65374$$

$$\ddot{a}_{60}^{00} \approx \bar{a}_{60}^{00} + \frac{1}{2} + \frac{\mu_{60}^{0\bullet} + \delta}{12} = 10.58355$$

$$\ddot{a}_{50:\overline{10}|}^{00} = \ddot{a}_{50}^{00} - e^{-10\delta}\,{}_{10}p_{50}^{00}\,\ddot{a}_{60}^{00} \approx 7.8199$$

(d)

EPV Death Benefit: $100{,}000\left(\bar{A}_{50}^{02} - e^{-10\delta}\,{}_{10}p_{50}^{00}\bar{A}_{60}^{02} - e^{-10\delta}\,{}_{10}p_{50}^{01}\bar{A}_{60}^{12}\right)$

$=100{,}000\left(0.21878 - e^{-0.5}(0.9088)(0.33174) - e^{-0.5}(0.04512)(0.41861)\right)$

$= 2240.14$

EPV CI Benefit: $50{,}000\left(\bar{A}_{50}^{01} - e^{-10\delta}\,{}_{10}p_{50}^{00}\bar{A}_{60}^{01}\right)$

$=50{,}000\left(0.21846 - e^{-0.5}(0.9088)(0.33692)\right)$

$= 1637.23$

EPV Premiums: $7.8199P$

$$\Rightarrow P = \frac{2240.14 + 1637.23}{7.8199} = 495.83$$

(e)

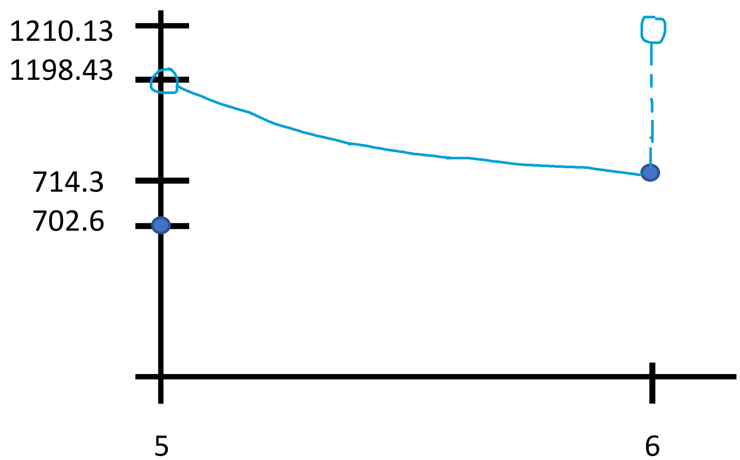
$$\left.\frac{d}{dt}\,{}_tV^{(0)}\right|_{t=5+} = \left({}_{5+}V^{(0)}\right)\delta - \mu_{55}^{01}\left(50{,}000 + {}_5V^{(1)} - {}_{5+}V^{(0)}\right) - \mu_{55}^{02}\left(100{,}000 - {}_{5+}V^{(0)}\right)$$

$${}_{5+}V^{(0)} = (702.6 + 495.8) = 1198.4$$

$$\Rightarrow \left.\frac{d}{dt}\,{}_tV^{(0)}\right|_{t=5+} = (1198.4)0.05 - 0.004265(50{,}000 + 5322.0 - 1198.4)$$

$$-0.004835(100{,}000 - 1198.4) = -648.6$$

(f)



Examiners' Comments

For Part (a), most candidate understood the features of each model; however very few candidates identified all the applicable models for each option and only picked one.
For part (b), most candidates got full credit, except a few put wrong integration limit or consider direct transition from state 0 to 2.

For Part (c), most candidate did well. Typical mistake was applying Woolhouse's formula to term annuity instead of whole-life annuity.

For Part (d), most candidate did well. Typical mistake was candidates omitted transition from state 1 to 2 in computing term life insurance.

For Part (e), a few candidates got full credit in this part. Most candidates didn't know how to apply Thiele's differentiation equation to the semi-continuous policy where premium is paid at the beginning of the year and death benefit is immediately upon transition. They didn't know where to put the premium payment at the beginning of the year in the Thiele's differential equation.

Most candidates didn't do well in this part (f) and didn't give details of the graph. They didn't realize the graph of reserve has stepwise jumps at premium payment times. Some sketched an increasing curve even though they computed a negative derivative in Part (e).

ALTAM F23 Q3

(a) The fund projection is as follows, where –/+ in the subscript indicates value before/after management charge deductions.

t	F_{t-}	MC	F_{t+}	F_{t+1-}
0	10000.0	200.0	9800.0	9800.0
1	9800.0	98.0	9702.0	9702.0
2	9702.0	97.0	9605.0	10085.2

Note: No need to put this in the form of a table.

(b) (i) The full profit test table is:

t	${}_{t-1}V$	MC_t	E_t	I_t	EDB_t	ESB_t	E_tV	Pr_t	Π_t	NPV
0			200.0				300.0	-500.0	-500.0	
1	300	200.0		15.0	49.0	-139.7	171.0	434.7	434.7	
2	200	98.0	58.8	7.2	48.5	0.0	85.5	112.4	96.1	
3	100	97.0	58.2	4.2	50.4	0.0	0.0	92.5	67.7	25.4

Key:

${}_{t-1}V$ is the reserve brought forward.

MC_t is the management charge/Unallocated Premium Income at the start of the t-th year. These are taken from the table in (a).

E_t denotes the expenses incurred at the start of the t-th year. The pre-contract expenses are assigned to year 0. The year 2 and 3 expenses are 0.006 F_{t-1-}

I_t denotes the interest earned on the insurer's funds. This is $0.03({}_{t-1}V + MC_t - E_t)$

EDB_t denotes the expected cost of death benefit in excess of the fund value. For this contract the DB is 110% of the fund value, so we have $EDB_t = q_{x+t-1}^{(d)}(0.10)F_{t-}$.

ESB_t denotes the expected cost of surrender or maturity benefit in excess of the fund value. For this contract, the SB is only 85% of the fund value in the first year, and the surrender probability in the first year is $q_x^{(s)} = (0.95)(0.10) = 0.095$, so that $ESB_1 = (0.095)(1-0.85)F_{1-}$

In the second year the SB is the fund value, so ESB_2=0

In the third year, the SB is the additional maturity benefit in excess of the terminal fund value, which is 0 in this projection.

E_tV denotes the expected cost of the reserve carried forward. In the $t=0$ row, this captures the cost of setting up the initial reserve of ${}_0V = 300$. In the first and second years, we have $E_tV = p_{x+t-1}^{(\tau)}\left({}_tV\right)$ where $p_{x+t-1}^{(\tau)} = 1 - 0.05 - (0.95)(0.10) = 0.855$ for $t = 0$ or 1. There is no cost in the third year.

Pr_t is the profit vector, Π_t is the profit signature.

OR Unsegregated version:

t	F_{t-1}	${}_{t-1}V$	E_t	If_t	I_t	EDB_t	ECV	E_tV	EF_t	Pr_t	Π_t
0			200.0					300.0		-500.0	-500.0
1	10000	300		0.0	15.0	539.0	791.4	171.0	8379.0	434.7	434.7
2	9800	200	58.8	0.0	7.2	533.6	921.7	85.5	8295.2	112.4	96.1
3	9702	100	58.2	480.2	4.2	554.7	9581.0	0.0	0.0	92.5	67.7

Where If_t is the interest earned on the policyholder's funds (after MC deduction) and I_t is interest earned on the other income, including the MC. The final payout could come under the CV or the EFt columns.

(ii) The NPV is $\sum_{t=0}^{3} \Pi_t v_{10\%}^t = 25.36$.

(c)

(i) Most policies continue to maturity. If the returns on policyholders' funds are poor, then all the policies maturing at the same time will require supplementary payments to meet the GMMB, and there will be no benefit from selling a larger number of policies. There is no diversification benefit.

(ii) Advantage: The GMMB will be hedged, so that the risk of very severe losses arising from a large number of policies maturing at the same time with fund values far below the initial investment is mitigated.

Disadvantage : The cost of hedging is large, relative to the potential income. This is a larger outlay than the reserve used in the profit test above, and unlike the reserve, the option cost is not recovered if the policy matures with no GMMB liability for the insurer. There is a very high likelihood of a loss on each policy.

Examiners' Comments

Part (a) was done well, with a large majority of candidates achieving maximum credit.

For Part (b), many candidates struggled to identify the correct cashflows to include in the profit test. The VA is a separate account contract, so that the policyholder's funds are not part of the cashflows involved in the profit test. The management charge and unallocated premium are included as income, and the outgo relevant to the insurer's funds only the difference between a claim/benefit payout and the amount of policyholder's funds available.

Part (c) was done moderately well. Most candidates who attempted the question recognized that a large part of the VA risk comes from poor investment returns, which is not diversified by selling more policies. The second part proved a bit more challenging, with many candidates failing to show an understanding of the costs and benefits of hedging the VA guarantees.

ALTAM F23 Q4

(a) The likelihood is the joint probability (density) function of the data. Each data point contributes its associated probability (density).

The probability associated with remaining in State 0 from age 70 to age 71 is

$$ {}_1p_{70}^{\overline{00}} = e^{-\int_0^1 \left(\mu_{70+t}^{01}+\mu_{70+t}^{03}\right)dt} = e^{-1\left(\mu_{70}^{01}+\mu_{70}^{03}\right)} $$

as the transition forces are constant between integer ages.

(b) The contribution is

$$ L_B = {}_{0.2}p_{70}^{\overline{11}}\,\mu_{70}^{10}\;{}_{0.3}p_{70}^{\overline{00}}\,\mu_{70}^{01}\;{}_{0.4}p_{70}^{\overline{11}}\,\mu_{70}^{13} $$

$$ = {}_{0.6}p_{70}^{\overline{11}}\;{}_{0.3}p_{70}^{\overline{00}}\,\mu_{70}^{10}\,\mu_{70}^{01}\,\mu_{70}^{13} $$

$$ = \exp\left\{-\left(0.6\left(\mu_{70}^{10}+\mu_{70}^{12}+\mu_{70}^{13}\right)+0.3\left(\mu_{70}^{01}+\mu_{70}^{03}\right)\right)\right\}\mu_{70}^{10}\,\mu_{70}^{01}\,\mu_{70}^{13} $$

(c) The MLE of μ_{70}^{01} is

$$ \widehat{\mu}_{70}^{01} = \frac{d_{70}^{01}}{t_{70}^{(0)}} = \frac{3}{0.9+0.3+0.3+0.5+6} = \frac{3}{8} = 0.375 $$

The numerator counts the 3 transitions from 0 to 1 (B, C, D) and the denominator counts the total exposure in State 0, i.e. Life A:0.9; Life B:0.3; Life C:0.3: Life D: 0.5, plus 6 other lives at 1 year each.

(d) Due to the invariance property of MLEs, the MLE for ${}_1p_{70}^{\overline{00}}$ is ${}_1\hat{p}_{70}^{\overline{00}} = e^{-\left(\hat{\mu}_{70}^{01}+\hat{\mu}_{70}^{03}\right)}$.

$$ \hat{\mu}_{70}^{03} = \frac{d_{70}^{03}}{t_{70}^{(0)}} = \frac{1}{0.9+0.3+0.3+0.5+6} = \frac{1}{8} = 0.125 $$

$$ \Rightarrow \hat{p}_{70}^{\overline{00}} = e^{-\left(\hat{\mu}_{70}^{01}+\hat{\mu}_{70}^{03}\right)} = e^{-0.5} = 0.6065 $$

(e) The SD of $\hat{\mu}_{70}^{01}$ is approximately $\dfrac{\sqrt{3}}{8} = 0.2165$

(f) This value of μ_{70}^{23} is zero since there were no transitions from State 2 to State 3 among the 20 lives being studied.

Examiners' comments

Part a: Successful candidates gave several reasons why (probability of staying in state 0, constant force, transition to states 1 or 3) and a brief discussion of each. Candidates generally did well here.

Part b: The most common error was candidates using actual ages instead of age 70 alone. A few also left answers in terms of probabilities and not transition intensities as the question asked.

Part c: Common errors here were candidates misunderstanding the total exposure, either by including too many individuals in state zero or using a full year time period instead of the actual time spent in state zero.

Part d: This was generally done well. The most common mistake was candidates misunderstanding which states applied (eg, some included transitions from zero to two despite it being impossible per the question).

Part e: The most common error was candidates providing the variance when the question asked for the standard deviation.

Part f: This was generally done well. There weren't many errors, but some were candidates either not providing enough detail or misunderstanding the question (assuming transition from state 3 to 2 instead of 2 to 3).

ALTAM Fall 2023 Q5

(a) Corridor factors exist to ensure that the additional death benefit (ADB) of a UL policy, over and above the account value, is large compared with the account value. Type A policies have a decreasing ADB, as it is the difference between the face value and the account value. For a long term policy, it is likely that the ADB becomes quite small as the account value grows. Type B policies have a constant ADB, so it is less likely that the ADB becomes too small relative to the account value. Hence, the Type A policy will be more likely to be impacted by the corridor factor than a Type B policy.

(b) AV_1 is less than P(1.07), so clearly the corridor factor does not apply.

$$CoI = q_{75} \cdot 100{,}000 \cdot v_{4\%} = 1772.40$$

(c)

$$AV_1 = (P - e - CoI)(1.07)$$

$$= (10{,}000(0.9) - 100 - 1772.40)(1.07) = 7626.53$$

(d) Again, AV_1 will be less than 10,700, and so the ADB will be greater than 89,300, and the corridor factor will not apply.

$$AV_1 = (P - e - CoI)(1.07)$$

$$= \left(P - e - q_{75}\, v_{4\%}\left(FA - AV_1\right)\right)(1.07)$$

$$\Rightarrow AV_1 = \frac{\left(P - e - q_{75}\, v_{4\%} FA\right)(1.07)}{1 - q_{75}\, v_{4\%}(1.07)}$$

$$= \frac{\left(10{,}000(0.9) - 100 - 100{,}000(0.018433)v_{4\%}\right)(1.07)}{1 - (0.018433)v_{4\%}(1.07)}$$

$$= 7773.96$$

(e) It is possible that the corridor factor will apply here. So AV_9 is the smaller of the AV ignoring the corridor factor, AV_9^f and the AV using the corridor factor, AV_9^c, where

$$AV_9^f = \frac{(AV_8+P-e-q_{83}v_{4\%}FA)(1.07)}{1-q_{83}v_{4\%}(1.07)} = 96{,}698.82$$

$$AV_9^c = (AV_8 + P - e - CoI)(1.07)$$

$$= (AV_8 + P - e - q_{83}v_{4\%}(cf-1)AV_9^c)(1.07)$$

$$= \frac{(AV_8 + P - e)(1.07)}{1 + q_{83}v_{4\%}(0.05)(1.07)}$$

$$= 96{,}626.45$$

$$\Rightarrow AV_9 = Min(96{,}698.82 \; and \; 96{,}626.45) = 96.626.45$$

Examiners' Comments

For part (a), a large number of candidates failed to articulate concisely that with a fixed total death benefit as the account value grows over time, the additional death benefit will decrease and therefore be more susceptible to the corridor adjustment.

For part (b), most candidates successfully calculated the cost of insurance, but many failed to demonstrate that the corridor adjustment was not applicable.

For Part (c), most candidates successfully calculated the account value of the policy at the end of the first policy year.

For part (d), most candidates were successful at calculating the account value of the policy at the end of the first policy year, but many failed to demonstrate that the corridor adjustment was not applicable.

For part (e), most candidates were able to calculate what the account value was without considering the corridor factor, some candidates failed to appropriately apply the corridor factor adjustment, which was relevant in this situation. Minimal points were awarded for failing to realize the corridor factor applied.

ALTAM Fall 2023 Q6

(a) Let r_{60x} denote the decrement table values for age retirements at exact age 60, and r_{60+}denotes values for age retirements between age 60 and 61, excluding exact age retirements.

(i)

$$AL_A = 0.02 \times FAS \times svc \left(v^{25} \frac{r_{60x}}{l_{35}} \ddot{a}_{60}^{(12)} + v^{25.5} \frac{r_{60+}}{l_{35}} \ddot{a}_{60.5}^{(12)} + v^{26} \frac{l_{61}}{l_{35}} \ddot{a}_{61}^{(12)} \right)$$

$$= 0.02 \times 60{,}000 \times 10 \begin{pmatrix} 1.05^{-25} \times \frac{27{,}925.61}{218{,}833.88} \times 14.441 \\ +1.05^{-25.5} \times \frac{6{,}187.56}{218{,}833.88} \times 14.315 \\ +1.05^{-26} \times \frac{58{,}699.91}{218{,}833.88} \times 14.186 \end{pmatrix}$$

$$= 12{,}000 \begin{pmatrix} 0.295303 \times 0.127611 \times 14.441 \\ +0.288186 \times 0.028275 \times 14.315 \\ +0.281241 \times 0.268240 \times 14.186 \end{pmatrix}$$

$$= 12{,}000(0.544193 + 0.116646 + 1.070190)$$

$$= 6{,}530.32 + 1{,}399.75 + 12{,}842.28$$

$$= 20{,}772.35$$

(ii)

$$AL_B = 0.02 \times FAS \times svc \left(v^{0.5} \frac{r_{60+}}{l_{60+}} \ddot{a}_{60.5}^{(12)} + v \frac{l_{61}}{l_{60+}} \ddot{a}_{61}^{(12)} \right)$$

$$= 0.02 \times 100{,}000 \times 30 \begin{pmatrix} 1.05^{-0.5} \times \frac{6{,}187.56}{65{,}159.77} \times 14.315 \\ +1.05^{-1} \times \frac{58{,}699.91}{65{,}159.77} \times 14.186 \end{pmatrix}$$

$$= 60{,}000 \begin{pmatrix} 0.975900 \times 0.094960 \times 14.315 \\ +0.952381 \times 0.900861 \times 14.186 \end{pmatrix}$$

$$= 60{,}000(1.326590 + 12.171064)$$

$$= 79{,}595.38 + 730{,}263.85$$

$$= 809{,}859.22$$

(b) The FAS for each life is the same as the current salary, as there have been no salary increases over the past 2 years.

(i) $NC_A = AL_A\left(\frac{FAS_{36}}{FAS_{35}}\frac{11}{10} - 1\right)$

$FAS_{36} = \frac{1}{2}(60{,}000 + 60{,}000 \times 1{,}1) = 63{,}000$

$NC_A = 20{,}772.35\left(\frac{63{,}000}{60{,}000} \times \frac{11}{10} - 1\right) = 3{,}219.71$

<u>or</u> (instead of shortcut admissible as there are no midyear exits)

$$NC_A = v\frac{l_{36}}{l_{35}}AL_A(t+1) + apv(\text{midyear benefits}) - AL_A(t)$$

$$= v\frac{l_{36}}{l_{35}}AL_A(t+1) - AL_A(t) \text{ (since there are no midyear benefits)}$$

$$FAS_{36} = \frac{1}{2}(60{,}000 + 60{,}000 \times 1{,}1) = 63{,}000$$

$$AL_A(t+1) = 0.02 \times FAS_{36} \times svc \times\left(v^{24}\frac{r_{60x}}{l_{36}}\ddot{a}_{60}^{(12)} + v^{24.5}\frac{r_{60+}}{l_{36}}\ddot{a}_{60.5}^{(12)} + v^{25}\frac{l_{61}}{l_{36}}\ddot{a}_{61}^{(12)}\right)$$

$$= 0.02 \times 63{,}000 \times 11\begin{pmatrix} 1.05^{-24} \times \frac{27{,}925.61}{207{,}871.81} \times 14.441 \\ +1.05^{-24.5} \times \frac{6{,}187.56}{207{,}871.81} \times 14.315 \\ +1.05^{-25} \times \frac{58{,}699.91}{207{,}871.81} \times 14.186 \end{pmatrix}$$

$$= 13{,}860\begin{pmatrix} 0.310068 \times 0.134341 \times 14.441 \\ +0.302595 \times 0.029766 \times 14.315 \\ +0.295303 \times 0.282385 \times 14.186 \end{pmatrix}$$

$$= 13{,}860(0.601535 + 0.128937 + 1.182958)$$

$$= 8337{,}28 + 1787{,}07 + 16{,}395.80$$

$$= 26{,}520.14$$

$$NC_A = 1.05^{-1} \times \frac{207{,}871.81}{218{,}833.88} \times 26{,}520.14 + 0 - 20{,}772.35 = 3{,}219.71$$

(ii) $NC_B = v\frac{l_{61}}{l_{60+}}AL_B(t+1) + apv(\text{midyear benefits}) - AL_B(t)$

$FAS_{61} = \frac{1}{2}(100{,}000 + 100{,}000 \times 1{,}1) = 105{,}000$

$$
\begin{aligned}
AL_B(t+1) &= 0.02 \times FAS_{61} \times svc \times \ddot{a}_{61}^{(12)} \\
&= 0.02 \times 105{,}000 \times 31 \times 14.186 \\
&= 65{,}100 \times 14.186 \\
&= 923{,}508.60
\end{aligned}
$$

$FAS_{60.5} = \frac{1}{2}(100{,}000 \times 1.5 + 100{,}000 \times 1{,}1 \times 0.5) = 102{,}500$

$$
\begin{aligned}
apv(\text{midyear benefits}) &= 0.02 \times FAS_{60.5} \times svc \times v^{0.5} \times \frac{r_{60+}}{l_{60+}} \times \ddot{a}_{60.5}^{(12)} \\
&= 0.02 \times 102{,}500 \times 30.5 \times 1.05^{-0.5} \times \frac{6{,}187.56}{65{,}159.77} \times 14.315 \\
&= 62{,}525 \times 0.975900 \times 0.094960 \times 14.315 \\
&= 62{,}525 \times 1.326590 \\
&= 82{,}945.01
\end{aligned}
$$

$$
\begin{aligned}
NC_B &= 1.05^{-1} \times \frac{58{,}699.91}{65{,}159.77} \times 923{,}508.60 + 82{,}945.01 - 809{,}859.22 \\
&= 65{,}422.07
\end{aligned}
$$

(iii) $NC\%_{agg} = \frac{NC_A + NC_B}{Sal_A + Sal_B} = \frac{3{,}219.71 + 65{,}422.07}{66{,}000 + 110{,}000} = 39.00\%$

(c) (i)

- The plan’s total AL for retirement benefits would stay the same.
- The AL values accrued benefits, and the new entrant has no past (pensionable) service and therefore no accrued benefits.

(ii)

- The aggregate normal contribution rate would decrease.
- The aggregate normal contribution rate is a weighted average of individual normal contribution rates, the weights being the individual salary rates. The new entrant’s normal contribution rate is lower than that of either current employee because he is age 35, equal to the lower of 35 and 60, and has no past service, unlike employees A and B.

Examiners' Comments

Actuarial liabilities were calculated well in general. The key issue was with using the Standard Service Table correctly for the person who had not retired at exact age 60.

For the normal cost, if a shortcut is used, the right one needs to be used – beware PUC versus TUC – and it needs to be applied correctly – TUC requires the FAS (not the annual salary) to be brought ahead one year. However, shortcuts do not work if there are midyear exits. It is advisable to master the first principles: they will work regardless of the method, whether there are midyear exits or not. Besides, midyear exits are meant to capture exits in the coming year, not all exits assumed to happen halfway through the year. Also, midyear exits do accrue 0.5 year in service and their FAS needs to be revised, just like the actuarial liability one year hence accrues 1 year in service along with a revised FAS.

When a question gives you a choice, it is wise to at least answer with one of the choices given. Then, the reasons have to match the choice! Comments on changes in absolute terms (in total actuarial liability) are generally well handled, whereas comments on changes in relative terms (in aggregate normal contribution rate) are generally weak. It helps to see the aggregate rate as a weighted average of individual rates. Many candidates correctly mentioned that the denominator increased, but the numerator increases as well. For new entrants, the actuarial liability is nil, but not the normal cost. To answer this question, it helps to remember that, with TUC, the normal cost as a % of salary increases a lot throughout a worker's career, as every additional year adds service and revalues all prior years of service.