LTAM Exam

Fall 2021

Solutions For Form A

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Solutions to Multiple Choice Questions

Fall 2021, Multiple Choice Solutions, Form A

MC1: Answer C

The cost of annual statements is an annual continuing expenses. It is not an initial expense.

MC2: Answer C

UDD: ${}_{\frac{1}{3}|\frac{1}{2}}q_0 = {}_{\frac{5}{6}}q_0 - {}_{\frac{2}{6}}q_0 = \left(\frac{5}{6} - \frac{2}{6}\right) \times 0.6 = 0.3$

CF: ${}_{\frac{1}{3}|\frac{1}{2}}q_0 = {}_{\frac{2}{6}}p_0 - {}_{\frac{5}{6}}p_0 = (0.4)^{\frac{2}{6}} - (0.4)^{\frac{5}{6}} = 0.27081$

Difference: $100,000(0.3 - 0.27081) = 100,000(0.02919) = 2919$

MC3: Answer E

$$\hat{S}(3) = 0.72 = \frac{50-10}{50} \times \frac{40+N-6}{40+N} \Rightarrow 0.72 = (0.8)\left(\frac{34+N}{40+N}\right) \Rightarrow 34 + N = 0.9(40+N)$$
$$\Rightarrow N = 20$$

MC4: Answer A

The probabilities are:

Zhanyi: $p_0^{00} \times p_1^{02} + p_0^{01} \times p_1^{12} = 0.8 \times 0.05 + 0.15 \times 0.1 = 0.055$

Yuanyuan: $p_0^{12} \times p_1^{12} = 0.9 \times 0.1 = 0.09$

Both: $0.055 \times 0.09 = 0.00495$

MC5: Answer B

$$q_{84}^{(1)} = q_{84}^{(\tau)} - q_{84}^{(2)} = \left(1 - \frac{9250}{10,000}\right) - 0.03 = 0.045$$

$$q_{85}^{(1)} = q_{84}^{(1)} + 0.015 = 0.045 + 0.015 = 0.06$$

$$d_{85}^{(\tau)} = l_{85}^{(\tau)} - l_{86}^{(\tau)} = 9250 - 8278 = 972 = d_{85}^{(1)} + d_{85}^{(2)} = 0.06 \times 9250 + d_{85}^{(2)} \Rightarrow d_{85}^{(2)} = 417$$

MC6: Answer C

Let $S = 100,000; \quad Z = Sv^{K_{35}+1}$

Z is a decreasing function of K_{35}, so the 98% quantile of Z corresponds to the 2% quantile of K_{35}. Let τ denote the 2% quantile of T_{35} :

$${}_{\tau}p_{35} = 0.98 \Rightarrow \frac{l_{55+\tau}}{l_{55}} = 0.98 \Rightarrow 7 < \tau < 8 \Rightarrow \text{2\% quantile of } K_{35} = 7$$

$$\Rightarrow \text{98\% quantile of } Z = Sv^{K_{35}+1} = Sv^{8} = 67,684$$

MC7: Answer B

$$EPV = 10,000\left({}_{2|1}q_{[75]}\, v^3 + {}_{3|1}q_{[75]}\, v^4 \right)$$
$$= 10,000\left(0.92 \times 0.9 \times 0.15 \times v^3 + 0.92 \times 0.9 \times 0.85 \times 0.2 \times v^4\right)$$
$$= 2,230$$

MC8: Answer E

$$EPV = \int_0^{\infty} {}_{t}p_x^{00}\left(400\mu^{(I)} + 700\mu^{(II)}\right)e^{-\delta t}dt$$
$$= \int_0^{\infty} e^{-0.08t}\left(400 \times 0.01 + 700 \times 0.02\right)dt = \frac{18}{0.08} = 225$$

MC9: Answer C

$$11.120 = 1000\frac{A_x}{\ddot{a}_x} = 1000\frac{A_x}{\left(\frac{1-A_x}{d}\right)} \Rightarrow A_x = 0.18931$$

$$76.529 = 1000\frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = 1000\frac{1-d\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \Rightarrow \ddot{a}_{x:\overline{n}|} = 8.0549$$

$$P = 1000\frac{0.18931}{8.0549} = 23.50$$

MC10: Answer D

Let $S = 100,000$ and $P = 1,045$

$$Z = \left(S + 500\right)v^{K_{45}+1} + 100 + 0.45P - 0.95P\ddot{a}_{\overline{K_{45}+1|}}$$

$$= \left(S + 500 + \frac{0.95P}{d}\right)v^{K_{45}+1} + 100 + 0.45P - \frac{0.95P}{d}$$

$$V[Z] = \left(S + 500 + \frac{0.95P}{d}\right)^2 \left({}^2A_{45} - A_{45}^2\right)$$

$$= \left(100,000 + 500 + \frac{0.95(1045)}{(0.05/1.05)}\right)^2 \left(0.03463 - (0.15161)^2\right) = 13,095^2$$

MC11: Answer B

Let τ denote the future lifetime. A profit is generated if $L < 0$ where

$$L = 100,000v^{\tau} + 100 - (5,280)(0.9)\left(\frac{1 - v^{\tau}}{\delta}\right)$$

$$\left(100,000 + \frac{(5280)(0.9)}{\ln(1.05)}\right) + 100 - \frac{(5280)(0.9)}{\ln(1.05)} < 0$$

$$197,397v^{\tau} < 97,297$$

$$\Rightarrow \tau > 14.4998$$

MC12: Answer A

Using Woolhouse's 2-term formula in a multiple state context, we have $\ddot{a}_x^{(m)ij} = \bar{a}_x^{ij}$ where $i \neq j$, and $\ddot{a}_x^{(m)ii} = \bar{a}_x^{ii} - \dfrac{1}{2m}$. Then

$$\begin{aligned}
\ddot{a}_{50:\overline{10}|}^{(2)01} &= \ddot{a}_{50}^{(2)01} - v^{10}\,{}_{10}p_{50}^{00}\,\ddot{a}_{60}^{(2)01} - v^{10}\,{}_{10}p_{50}^{01}\,\ddot{a}_{60}^{(2)11} \\
&= \bar{a}_{50}^{01} - v^{10}\,{}_{10}p_{50}^{00}\,\bar{a}_{60}^{01} - v^{10}\,{}_{10}p_{50}^{01}\left(\bar{a}_{60}^{11} + \frac{1}{4}\right) \\
&= 1.9621 - (1.05)^{-10}(0.83936)(2.6295) - (1.05)^{-10}(0.06554)(10.2084 + 0.25) \\
&= 0.18633
\end{aligned}$$

So, $2P(7.5385) = 2 \times 5000 \times 0.186331 \Rightarrow P = 123.59$

MC13: Answer B

EPV Premiums: $P\ddot{a}_{50:50:\overline{10}|} = 8.0027P$

EPV Benefits: $400{,}000 A^{\;1}_{\overline{50:50}:\overline{10}|} + 60{,}000\ {}_{10}E_{50:50}\ \ddot{a}_{\overline{60:60}}$

$${}_{10}E_{50:50} = {}_{10}E_{50} \cdot {}_{10}p_{50} = (0.60182)\left(\frac{96{,}634.1}{98{,}576.4}\right) = 0.58996$$

$$\ddot{a}_{\overline{60:60}} = \ddot{a}_{60} + \ddot{a}_{60} - \ddot{a}_{60:60} = 14.9041 + 14.9041 - 13.2497 = 16.5585$$

$$(400{,}000)(0.02896) + (60{,}000)\,(0.58996)\,(16.5585) = 597{,}715$$

$$\Rightarrow P = \frac{597{,}715}{8.0027} = 74{,}689$$

MC14: Answer D

$$P\ddot{a}_{50} = 1000A_{50} + 30\ddot{a}_{50} + 270 + 0.04P\ddot{a}_{50} + 0.26P$$

$$P = \frac{1000A_{50} + 30\ddot{a}_{50} + 270}{0.96\,\ddot{a}_{50} - 0.26} = \frac{1000(0.18931) + 30(17.0245) + 270}{0.96(17.0245) - 0.26} = 60.31$$

$$\left({}_0V + P \times 0.7 - 300\right)(1+i) = q_{50} \times 1000 + p_{50} \times {}_1V$$

$$\Rightarrow {}_1V = \frac{(0 + (0.7)(60.31) - 300)(1.05) - (0.001209)(1000)}{1 - 0.001209} = -272.21$$

MC15: Answer B

$$\ddot{a}_{36:\overline{24|}} = 1 + p_{36} \cdot v \cdot \ddot{a}_{37:\overline{23|}} = 1 + (0.99959)(1.05)^{-1}(16.8078) = 17.3116$$

$${}_1V = (100,000)\left(1 - \frac{\ddot{a}_{37:\overline{23|}}}{\ddot{a}_{36:\overline{24|}}}\right) = (100,000)\left(1 - \frac{16.8078}{17.3116}\right) = 2910$$

or

$$A_{36:\overline{24|}} = 1 - d\,\ddot{a}_{36:\overline{24|}} = 1 - (0.05/1.05)(17.3116) = 0.49578$$

$$P = (100,000)\left(\frac{A_{36:\overline{24|}}}{\ddot{a}_{36:\overline{24|}}}\right) = \frac{(100,000)(0.49578)}{17.3116} = 2863.86$$

$$({}_0V + P)(1.03) = (100,000)q_{36} + p_{36} \cdot {}_1V$$

$${}_1V = \frac{(0 + 2863.86) - (100,000)(0.00041)}{1 - 0.00041} = 2910$$

MC16: Answer A

$${}_{20}V = SA^{\ 1}_{70:70} - P\ddot{a}_{70:70}; \quad A^{\ 1}_{70:70} = \tfrac{1}{2}A_{70:70}$$

$$\Rightarrow {}_{20}V = (1,000,000)(0.5)(0.52488) - (7797)(9.9774) = 184,646$$

MC17: Answer E

$$V = 12\times 3,500\times \ddot{a}_{75}^{(12)00} + 12\times 8,000\times \ddot{a}_{75}^{(12)01} + 12\times 20,000\times \ddot{a}_{75}^{(12)02}$$
$$+ 50,000\times \bar{A}_{75}^{03} - 12\times 4,000\times \left(\ddot{a}_{75}^{(12)00} + \ddot{a}_{75}^{(12)01} + \ddot{a}_{75}^{(12)02}\right)$$

$$= (12)(3,500)(8.751) + (12)(8,000)(0.754) + (12)(20,000)(0.397)$$
$$+ (50,000)(0.178) - (12)(4,000)\left(8.751 + 0.754 + 0.397\right)$$

$$= 68,810$$

MC18: Answer C

$$NPV = -350 + 125v + 130v^2 + 135v^3 + 140v^4 + 145v^5 \quad \text{at } i = 0.12$$
$$= 132.58$$

$$PM = 0.0145 = \frac{132.58}{P\ddot{a}_{50:\overline{5}|}} \Rightarrow P = \frac{132.58}{(0.0145)(4.0278)} = 2270$$

MC19: Answer D

$$FAS = 80,000$$
$$AL = 0.02\times 80,000\times 12\times {}_{20}E_{45}\times 14.3714 = 120,627$$

MC20: Answer E

$$EPV = 3000\,\ddot{a}_{70} \quad \text{at} \quad i^* = \frac{1+i}{c(1+j)} - 1 = -0.03$$

$$\Rightarrow EPV = (3000)(27.1070) = 81,321$$

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Fall 2021

Solutions to Written Answer Questions

Question 1 Model Solution

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a)

For T_{50}=11.8, K_{50}=11 and 12 premiums would be returned without interest.

S=100,000 and P=2000.

$$L^g = (S + 12P + 1000)v^{12} + 0.75\,P - 0.95\,P\,\ddot{a}_{\overline{12|}}$$

$$\ddot{a}_{\overline{12|}} = \frac{1 - 1.05^{-12}}{0.05/1.05} = 9.306414$$

$$L^g = (100{,}000 + 12(2000) + 1000)\,1.05^{-12} + 0.75(2000) - 0.95(2000)(9.306414)$$

$$L^g = 69{,}604.6773 + 1500 - 17{,}682.187 = 53{,}422.49$$

Comments: Candidates generally did well in this part. Common mistakes were using 11.8 or 11 as the number of premiums or not properly including the return of premium benefit.

b)

$$E[L^g] = (S + 1000)A_{50} + (P\ {}_{10}E_{50})(10\,A_{60} + (IA)_{60}) \qquad + 0.75\,P - 0.95\,P\,\ddot{a}_{50}$$

$$E[L^g] = (S + 1000)A_{50} + \underbrace{(10P)\,{}_{10}E_{50}\,A_{60} + P\ {}_{10}E_{50}(IA)_{60}}_{\text{RoP:}} + 0.75\,P - 0.95\,P\,\ddot{a}_{50}$$

$$= (101{,}000)(0.18931) + (20{,}000)(0.60182)(0.29028) + (2000)(0.60182)(6.63303) + (0.75)(2000) - (0.95)(2000)(17.0245)$$

$$= 32{,}098.016 - 32{,}346.55 = -248.53$$

Comments: Candidates generally struggled to properly including the return of premium benefit.

c)

$$ {}_{10}V^g = (S+1000)A_{60} + 10P\,A_{60} + P(IA)_{60} - 0.95\,P\,\ddot{a}_{60} $$

$$ {}_{10}V^g = (100{,}000+1000)(0.029028) + 10(1000)(0.229028) + (2000)(6.63303) $$
$$ -0.95(2000)(14.9041) $$

= 29,318.28 + 5805.60 + 13,266.06 – 28,317.79 = 20,072.15

Comments: Candidates did better on this part than on Part (b) but many still struggled to properly including the return of premium benefit.

d)

(i) Total gain: using the actual experience (*) during year 16,

$$ 1000({}_{15}V^g + P - e_{15}^*)(1+i_{15}^*) - 1000\,q_{65}^*(S+16P+E_{15}^*) - 1000\,p_{65}^*\,{}_{16}V^g $$

=1000(34,333.78+0.96(2000))(1.052)–7(100,000+32,000+2000) – 993(37,480.51)

= 38,138,976.56 – 938,000 – 37,218,146.43

= –17,169.87 i.e. a loss of 17,169.87

(ii) Gain by source:

Gain from expenses (E):

Expected expenses – Actual expenses, valued at year end

$$ [1000(0.05P)(1.05) + 1000\,q_{65}(1000)] - [1000(0.04P)(1.05) + 1000q_{65}(2000)] $$

= [105,000 + 1000(0.005915)(1000)] – [84,000 + 1000(0.005915)(2000)]

= 110,915 – 95,830 = **15,085** ➔ a gain from expenses of 15,085

Alternatively, using actual experience for expenses only, total gain would have been

$$ 1000({}_{15}V^g + P - e_{15}^*)(1.05) - 1000q_{65}(S+16P+E_{15}^*) - 1000\,p_{65}\,{}_{16}V^g $$
$$ = 1000\big(34{,}333.78 + 0.96(2000)\big)(1.05) - 5.915(100{,}000+32{,}000+2000) $$
$$ -994.085\,(37{,}480.51) $$

$$ = 38{,}066{,}469 - 792{,}610 - 37{,}258{,}812.78 = \mathbf{15{,}046.22} $$

Since the expected gain is

$$1000\big(34{,}333.78 + 0.95(2000)\big)(1.05) - 5.915(100{,}000 + 32{,}000 + 1000)$$
$$-994.085\ (37{,}480.51) = -38.78$$

the <u>gain</u> from expenses is 15,046.22 + 38.78 = **15,085**.

<u>Gain from interest (I)</u>:

Actual interest earned – expected interest earned, using the actual expenses

$1000({}_{15}V^g + 0.96P)(i_{15}^* - 0.05)$= 1000 [34,333.78 + 0.96(2000)] (.002)

= **72,507.56**➔a gain from interest of 72,507.56

Alternatively, using actual expenses and interest rate, the gain would have been

$1000({}_{15}V^g + P - e_{15}^*)(1.052) - 1000\ q_{65}(S + 16P + E_{15}^*) - 1000\ p_{65}\ {}_{16}V^g$

= 38,138,976.56 – 792,610 – 37,258,812.78 = 87,553.78

So the gain from interest is 87,553.78 – 15,046.22 = 72,507.56

<u>Gain from mortality (M)</u>:

Expected mortality cost – actual mortality cost, using the actual expenses and interest

$$[1000q_{65}(S + 16P + E_{15}^*) + 1000p_{65}\ {}_{16}V^g]$$
$$-[1000\ (0.007)(S + 16P + E_{15}^*) + 1000\ (0.993)\ {}_{16}V^g]$$

= 792,610 + 37,258,812.78 – 938,000 – 37,218,146.43 = **–104,723.65**

➔ a loss from mortality of 104,723.65

Alternatively, the total gain calculated in (c) is –17,169.87.

So, the gain from mortality is –17,169.87 – 87,553.78 = –104,723.65

➔a loss from mortality of 104,723.65

Comments:

- *For part (d), gain by interest was done the best while gain by mortality was done the worst.*
- *For part (d), some candidates confused when to use actuals vs. expected in the gain by source analysis.*
- *Some candidates answered part (d).ii before part (d).i and used the sum of the gains from part (d).ii to answer part (d).i, if done correctly, this received full credit.*
- *Some candidates forgot to multiply their gains by 1000 policies - a small deduction was made for this.*

e)

The orders **MEI and EMI** had the same interest gain as EIM.

This can be justified a number of ways.

- The mortality cost is incurred at year end and has no impact of the interest earned during the year.
- Since actual premium expenses are different than expected, the gain from expenses (E) must be calculated before that from interest (I) to get the same value. So, E must precede I.
- Expenses affect the interest earned, so E must be calculated before I.

Comments: Some candidates mistakenly commented on the total gain (i.e. stated that regardless of the order, the total gain will not change) - this did not receive any credit.

Question 2 Model Solution

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a)

$Pr_0 = -3000 - (0.15)\,P = -3000 - (0.15)(80{,}000) = -15{,}000$

Comments: Candidates generally did well here. Common mistakes included calculating a reserve when the question specifically stated V=0, recalculating P when the question explicitly gave a gross premium, or calculating the actuarial present value instead.

b)

$Pr_2 = (0.95P - 100)(1.07) - S\,q_{x+1}$

$= (0.95(80{,}000) - 100)(1.07) - 1{,}000{,}000\,(0.1)$

$= 81{,}213 - 100{,}000 = -18{,}787$

Comments: Candidates generally did well here. The mistakes that were made were the same as those made in Part a. Additionally, some candidates also incorrectly applied survival probabilities that should have otherwise been applied in part c.

c)

$$\begin{aligned}\Pi &= (-15{,}000;\ 31{,}213;\ \ Pr_2\ {}_1p_x^{(\tau)}) \\ &= (-15{,}000;\ \ 31{,}213;\ \ -18{,}787\ (1-0.05)(.9)) \\ &= (-15{,}000;\ 31{,}213;\ -16{,}062.885)\end{aligned}$$

*Comments: The most common mistake here was not treating the decrements as independent for the profit signature at time 2, ie calculating 1 - q - lapse instead of (1-q) * (1 - lapse). Many candidates also forgot to include one of the two decrements entirely.*

d)

NPV = $-15{,}000 + 31{,}213\, v_{0.2} - 16{,}062.885\, v_{0.2}^2$
= – 15,000 + 26,010.83 – 11,154.78 = –143.95

Comments: Candidates were generally able to correctly calculate an NPV

e)

Let *j*=IRR

$-15{,}000 + 31{,}213\, v_j - 16{,}062.885\, v_j^2 = 0$

$\Rightarrow\ v_j = \frac{31{,}213 \pm \sqrt{31{,}213^2 - 4(15{,}000)(16{,}062.885)}}{2(16{,}062.885)} = \frac{31{,}213 \pm 3237}{32{,}125.77}$

$\Rightarrow\ \boldsymbol{j} = -6.7\%\ or\ \mathbf{14.83}\%$ ➔ Keeping the positive rate, IRR= 14.83%.

Alternatively,

$-15{,}000 + 31{,}213\,(1+j)^{-1} - 16{,}062.885\,(1+j)^{-2} = 0$

$-15{,}000(1+j)^2 + 31{,}213\,(1+j) - 16{,}062.885 = 0$

$$1 + j = \frac{31{,}213 \pm \sqrt{31{,}213^2 - 4(15{,}000)(16{,}062.885)}}{2(-15{,}000)} = \frac{31{,}213 \pm 3237}{-30{,}000}$$

$\boldsymbol{j} = -6.7\%\ or\ \mathbf{14.83}\%$.

Comments: The majority of candidates knew the formula for an IRR. Candidates were most often tripped up in cases where mistakes carried forward from prior parts made it impossible to calculate an IRR, in particular when the quadratic was negative.

f)

The NPV is being calculated at a risk discount rate of 20% while the IRR uses a discount rate of 14.83. The 20% discount rate results in a negative because the discounted profits at the end of one year and at the end of two years when discounted at 20% are not sufficient to cover the pre-contract expenses. The NPV is positive for some rates of return less than 20%. The IRR>0 for discount rates less than IRR=14.83% (rates between -6.7% and 14.83%).

Comments: Many candidates simply restated the question here and could not point to how it's possible to have a positive IRR but a negative NPV. Some mentioned the difference in discount rate vs IRR, but we were really looking for commentary of the timing of the cash flows. Few candidates got full marks here.

g)

With a 15% lapse rate, ${}_1p_x^{(\tau)} = (1 - 0.05)(.85) = 0.8075$

The profit signature becomes

$\Pi = (-15{,}000;\ 31{,}213;\ Pr_2\ {}_1p_x^{(\tau)})$

$= (-15{,}000;\ 31{,}213;\ -18{,}787\ (1-0.05)(.85))$

$= (-15{,}000;\ 31{,}213;\ -15{,}170.50)$

And NPV at 20% is

NPV = $-15{,}000 + 31{,}213\ v_{0.2} - 15{,}170.50\ v_{0.2}^2$

= – 15,000 + 26,010.83 – 10,535.07 = 475.76

Comments: Candidates performed here very similarly to C, and mistakes there related to decrements often carried over here.

h)

If **lapses be lower** than expected in the pricing, the insurer would make **less profit** than expected, and possibly even a loss. Lapse rates are unpredictable. When the policyholder is better off not lapsing, it may result in lower lapse rates. Lower lapse rates could easily produce losses for the company. In the market today, policyholders have the option to sell their policies to third parties. The third party will pay the policyholder for the policy (compared to a cash value of zero if the policy is lapsed) which may result in very low lapses.

Comments: Candidates struggled on this question as well. Many answered either the opposite of what was asked or something completely unrelated. Very few were able to point out that the decision to lapse is the policyholder's, and there are many factors outside the control of the insurer to impact it.

Question 3 Model Solution

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a)

(i)

$$\overline{A}_0^{03} = \int_0^\infty {}_tp_0^{01}\ \mu_t^{13}\ e^{-\delta t}\ dt$$

$$\overline{A}_0^{03} = \int_0^\infty {}_tp_0^{01}\ \mu_0^{13}\ e^{-\delta t}\ dt$$

$$\overline{A}_0^{03} = \mu_0^{13} \int_0^\infty {}_tp_0^{01}\ \ e^{-\delta t}\ dt$$

since μ_t^{13} is constant for all t

$$\overline{a}_0^{01} = \int_0^\infty {}_tp_0^{01}\ \ e^{-\delta t}\ dt$$

⇨ $\overline{A}_0^{03} / \overline{a}_0^{01} = \mu_0^{13}$

(ii)

$$\overline{a}_t^{\overline{11}} = \int_0^\infty {}_rp_t^{\overline{11}}\ \ e^{-\delta r}\ dr$$

Where ${}_rp_t^{\overline{11}} = e^{-\int_0^r (\mu_{t+s}^{12} + \mu_{t+s}^{13})\ ds} = e^{-\int_0^r 0.2+0.1)\ ds} = e^{-0.3\ r}$

$$\overline{a}_t^{\overline{11}} = \int_0^\infty e^{-0.3\ r}\ \ e^{-0.04\ r}\ dr = \int_0^\infty e^{-0.34\ r}\ dr = \frac{1}{0.34} = 2.941$$

(iii)

$$\overline{a}_t^{\overline{11}} < \overline{a}_t^{11}$$

since $\overline{a}_t^{11}$ also includes the (positive) EPV of payments made on returning to state 1 after visiting state 2.

or

$$\overline{a}_t^{11} = \overline{a}_t^{\overline{11}} + \underbrace{\int_0^\infty {}_rp_t^{12}\ \mu_{t+r}^{21}\ \overline{a}_{t+r}^{\overline{11}}\ e^{-\delta r}\ dr}_{>0}$$

⇨ $\overline{a}_t^{11} > \overline{a}_t^{\overline{11}}$

Or

$\overline{a}_t^{11} = \int_0^\infty {}_rp_t^{11}\ \ e^{-\delta r}\ dr$ and $\overline{a}_t^{\overline{11}} = \int_0^\infty {}_rp_t^{\overline{11}}\ \ e^{-\delta r}\ dr$

Since ${}_rp_t^{11} > {}_rp_t^{\overline{11}}$, we have $\overline{a}_t^{11} > \overline{a}_t^{\overline{11}}$

Comments: Most candidates completed Part i correctly. For Part ii, most candidates jumped straight to the memorized shortcut. For all parts, some candidates had minor expression errors where their variables and/or integral limits were not consistent.

Almost all candidates were able to correctly justify that annuity factor $\overline{a}_t^{\overline{11}}$ is smaller than $\overline{a}_t^{11}$.

b)

(i) $22{,}000\,\overline{A}_0^{03} = 22{,}000\,(0.1)\,\overline{a}_0^{01}$ from (a) (i)

$= 22{,}000\ (0.1)\ (2.930) = 6446$

(ii) $12{,}000\,\overline{a}_0^{01} = 12{,}000\ (2.930) = 35{,}160$

Comments: Most candidates were able to get the correct answers for parts (i) and (ii).

(iii) EPV is the sum of EPV of additional costs during first sojourn in State 1 (from State 0) up to 4 months, plus the EPV of additional costs during returning sojourns in State 1 from State 2 up to 4 months.

EPV of 1st sojourn   EPV of returning sojourns

$$8000\left\{\int_0^{\infty} {}_tp_0^{00}\,\mu_t^{01}\,\overline{a}_{t:\overline{1/3}|}^{\overline{11}}\,e^{-\delta t}\,dt \quad + \quad \int_0^{\infty} {}_tp_0^{02}\,\mu_t^{21}\,\overline{a}_{t:\overline{1/3}|}^{\overline{11}}\,e^{-\delta t}\,dt \right\}$$

Where $\overline{a}_{t:\overline{1/3}|}^{\overline{11}}$ is constant for all t

$$\overline{a}_{t:\overline{1/3}|}^{\overline{11}} = \int_0^{1/3} {}_rp_t^{\overline{11}}\ e^{-\delta r}\,dr$$

$$= \int_0^{1/3} e^{-0.3\,r}\ \ e^{-0.04\,r}\,dr = \frac{1-e^{-0.34\,(1/3)}}{0.34} = 0.3151$$

Alternatively,

$$8000\left\{\int_0^{\infty} {}_tp_0^{00}\,\mu_t^{01}\int_0^{1/3} {}_rp_t^{\overline{11}}\;\; e^{-\delta r}\,dr\;e^{-\delta t}\,dt\;+\right.$$
$$\left.\int_0^{\infty} {}_tp_0^{02}\,\mu_t^{21}\int_0^{1/3} {}_rp_t^{\overline{11}}\;\; e^{-\delta r}\,dr\;e^{-\delta t}\,dt\;\right\}$$

Comments: Only a handful candidates were able to answer this correctly. Most candidates received 0 marks for this question.

c)

The expected time of the first transition to State 1 is unchanged.

i) **Decrease**

With a larger value of μ_t^{13}, the **expected time spent in State 1 will decrease.** With less time spent in State 1, the EPV of the cost in nursing care in State 1 will **decrease**.

ii) **Increase**

With a larger value of μ_t^{13}, the transition to State 3 **(surgery) will happen sooner, on average**. With surgery happening earlier, the EPV of the cost of surgery will **increase**.

or

Since $\overline{A}_0^{03} = \int_0^{\infty} {}_tp_0^{01}\;\mu_t^{13}\,e^{-\delta t}\,dt$, changing only μ_t^{13} to a larger value will increase $\overline{A}_0^{03}$.

Comments: For part (i), most candidates were able to answer and justify that higher μ_t^{13} will cause higher probability (and thus shorter time) from State 2 to State 3. For part (ii), while most candidates were able to correctly answer that a higher μ_t^{13} will increase EPV of surgery, they were not able to clearly express that the increased cost was due to shorter travel time to State 3 (absorbing state) and thus the EPV with less discounting is higher. Some incorrect justifications included "higher probability for surgery" or "more surgery" and etc.

Question 4 Model Solution

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a)

$e_x = \sum_{k=1}^{\infty} {}_kp_x$

$= \sum_{k=1}^{n} {}_kp_x + \sum_{k=n+1}^{\infty} {}_kp_x$

$= \sum_{k=1}^{n} {}_kp_x + \sum_{k=1}^{\infty} {}_np_x \; {}_kp_{x+n} \; = \sum_{k=1}^{n} {}_kp_x + {}_np_x \sum_{k=1}^{\infty} \; {}_kp_{x+n}$

$= e_{x:\overline{n}|} + {}_np_x \; e_{x+n}$

Comments: Candidates did well on this part with many earning full credit. A common mistake is that candidates added ${}_kp_x$ from k=0 instead of 1. Several candidates attempted to explain the formula verbally or prove it for a special case (e.g., when n=3) rather than mathematically prove it for general n. Some candidates attempted to prove the continuous analogy of the given formula.

b)

$e_{87} = e_{87:\overline{3}|} + {}_3p_{87} \; e_{90} = \; 6.56$

$e_{87:\overline{3}|} = \sum_{k=1}^{3} {}_kp_{87} = \; 0.927763 + \; 0.852802 \; + 0.775771 = 2.556336$

$e_{90} = (6.56 - 2.556336)/0.775771 \; = 5.1609$

Alternatively, candidates may calculate using the recursive formula three times:

$e_{87} = p_{87} \, (e_{88} + 1)$ ➔ $e_{88} = \frac{6.56}{0.927763} - 1 = 6.07077$

$e_{88} = p_{88} \, (e_{89} + 1)$ ➔ $e_{89} = \frac{6.07077}{0.919202} - 1 = 5.60439$

$e_{89} = p_{89} \, (e_{90} + 1)$ ➔ $e_{90} = \frac{5.60439}{0.909674} - 1 = 5.16$

Comments: Candidates generally did well on this part. Similar to a), when calculating the 3-year curtate life expectancy, some candidates mistakenly added ${}_kp_{87}$ from k=0 instead of 1.

c)

$\text{V[H]} = \text{E}[H^2] - (e_{87:\overline{3}|})^2$

$\text{E}[H^2] = \sum_{k=1}^{3}(2k-1)\ {}_{k}p_{87}$

$= 1 \times 0.927763 + 3 \times 0.852802 + 5 \times 0.775771 = 7.365021169$

Alternatively

$$\begin{aligned}\text{E}[H^2] &= \sum_{k=0}^{2} k^2\ {}_{k|}q_{87} + 3^2\ {}_{3}p_{87} \\ &= 1^2\ (0.927763)(0.080798) + 2^2\ (0.852802)(0.090326) + 3^2\ (0.775771) \\ &= 0.07496139487 + 0.3081207738 + 6.981939 = 7.365021169\end{aligned}$$

$\text{V[H]} = 7.365021169 - (2.556336)^2 = 0.8301674$

SD[H] = 0.9111

Comments: Candidates did fair on this part. Various mistakes were found in calculating the second moment. For examples, many candidates mistakenly shifted the value of H by 1 (e.g., they started H=1 with probability q_{87} rather than H=0). Additionally, some candidates incorrectly remembered the formula for the second moment formula $\sum_{k=1}^{3}(2k-1)\ {}_{k}p_{87}$ using 2k or (2k+1) instead of (2k-1).

d)

(i)

$$\begin{aligned}\tilde{e}_{87:\overline{3}|} &= \sum_{k=1}^{3} {}_{k}\tilde{p}_{87} \\ &= \big(1 - q(87{,}0)\big) + \big(1 - q(87{,}0)\big)\big(1 - q(88{,}1)\big) \\ &\quad + (1 - q(87{,}0))(1 - q(88{,}1))(1 - q(89{,}2)) \\ &= 0.927763 + 0.927763\big(1 - 0.080798(0.978)\big) \\ &\quad + 0.927763(1 - 0.080798(0.978))(1 - 0.090326(0.979)^2) \\ &= 0.927763 + 0.854451 + 0.780479 = 2.562693\end{aligned}$$

(ii)

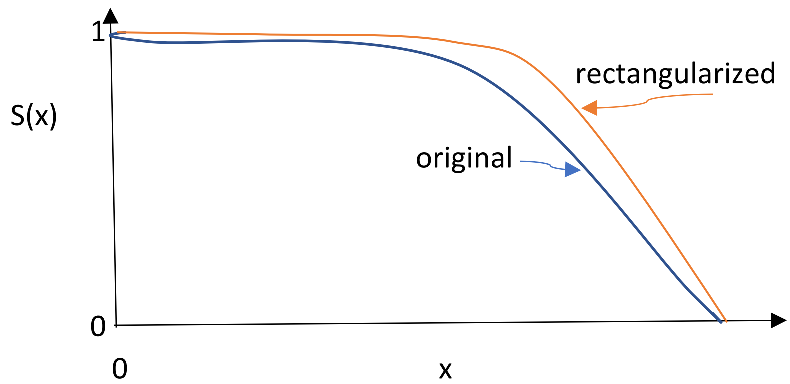
$\tilde{e}_{87} = \tilde{e}_{87:\overline{3}|} + {}_{3}\tilde{p}_{87}\ \tilde{e}_{90} = \tilde{e}_{87:\overline{3}|} + {}_{3}\tilde{p}_{87}\ e_{90}$

$\tilde{e}_{87} = 2.562693 + (0.780479)(5.1609) = 6.5907$

Comments: Candidates did poor on this part. Many mistakes were made in identifying the proper mortality improvement factors. A few candidates added an improvement factor for q(87, 0), while there should be no mortality improvement. Some candidates incorrectly used q(87,1) and q(87,2) for q(88,1) and q(89,2), respectively.

e)

(i)



Mortality improves such that more people live to older ages but not much improvement in the oldest attainable age. The survival curve has more of a rectangular shape without shifting its right part much.

(ii) The mortality improvement scale **will cause a rectangularization** of mortality. The mortality rates for ages up to 90 will decrease causing more people to live to older ages. With no improvement in mortality rates after age 90, the right tail of the survival curve will not be affected much.

Comments: Candidates did very poor on this part with majority earning no credit. It appears that candidates didn't understand what "rectangularization of mortality" means.

Question 5 Model Solution

(a)

$$200{,}000\, A_{50:60:\overline{10|}} = P\, \ddot{a}_{50:60:\overline{10|}}$$
$$\ddot{a}_{50:60:\overline{10|}} = 7.9044$$
$$A_{50:60:\overline{10|}} = 1 - d\, \ddot{a}_{50:60:\overline{10|}} = 1 - \frac{0.05}{1.05}(7.9044) = 0.6236$$

Or

$$A_{50:60:\overline{10|}} = A_{50:60} + v^{10}\,{}_{10}p_{50}\,{}_{10}p_{60}\,(1 - A_{60:70}) = 0.6236$$

$v^{10}\,{}_{10}p_{50}\,{}_{10}p_{60}$ = (0.613913)(96,634.1/98,576.4)(91,082.4/96,634.1)
= (0.613913)(0.9802965)(0.942549) = 0.5672418

$A_{50:60} = 0.32048$ $\quad\quad$ $A_{60:70} = 0.46562$

$$A_{50:60:\overline{10|}} = 0.32048 + 0.5672418(1 - 0.46562) = 0.6236$$

$$P = \frac{(200{,}000)A_{50:60:\overline{10|}}}{\ddot{a}_{50:60:\overline{10|}}} = \frac{(200{,}000)(0.6236)}{7.9044} = 15{,}778.55$$

Or

$$P = (200{,}000)\left(\frac{1}{\ddot{a}_{50:60:\overline{10|}}} - d\right) = (200{,}000)\left(\frac{1}{7.9044} - \frac{0.05}{1.05}\right) = 15{,}778.55$$

Comments: Candidates did very well on this part. The most common issue was pulling the wrong values for the tables or incorrectly calculating the annuity factor which could be pulled directly from tables.

(b)

$${}^2A_{50:60:\overline{10|}} = {}^2A_{50:60} + v^{20}\,{}_{10}p_{50}\,{}_{10}p_{60}\,(1 - {}^2A_{60:70})$$
$$= {}^2A_{50:60} + {}_{10}E_{50}\,{}_{10}E_{60}\,(1 - {}^2A_{60:70})$$
$$= 0.12929 + (0.60182)(0.57864)(1 - 0.24895)$$

= 0.3908335

Comments: Candidates also did well on this part. Common errors were missing the endowment part of the equation or if they had the wrong discount factors for the endowment.

(c)

$$Var[L] = \left(S + \frac{P}{d}\right)^2 \ ({}^2A_{50:60:\overline{10|}} - (A_{50:60:\overline{10|}})^2)$$

$$= (531{,}349.55)^2(0.390834 - 0.6236^2) = (531{,}349.55)^2(0.001957)$$

$$= 552{,}524{,}514.1 = 23{,}505.84^2$$

Standard Deviation of L = 23,505.84

Comments: Overall, candidates did well on this part also. Many candidates have this formula memorized. Partial credit was awarded if there was an attempt to derive the formula, if it wasn't memorized.

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(d)

$$G = \frac{S \ A_{50:60:\overline{10|}}}{(0.9)\ddot{a}_{50:60:\overline{10|}} - 0.15} = \frac{124{,}720}{6.96396} = \mathbf{17{,}909.35}$$

Comments: Again, candidates did well on this part. The most common error was the double counting first year expense (using a separate 0.25 instead and using .15)

(e)

(i) $G^* = \frac{S\ {}^2A_{50:60:\overline{10}|}}{(0.9)\ {}^2\ddot{a}_{50:60:\overline{10}|} - 0.15} = \frac{200{,}000(0.390834)}{5.747024} = \mathbf{13,601.27}$

$${}^2\ddot{a}_{50:60:\overline{10}|} = \frac{1 - {}^2A_{50:60:\overline{10}|}}{d^*} = 6.552249$$

$$d^* = 0.1025/1.1025 = 0.0929705$$

A reduction of (17,909.35 − 13,601.27)/17,909.35 = 0.2405 or **24.05%**

(ii) The percentage reduction would be **less** for a 10-year term insurance. The endowment insurance also pays a benefit in case of survival to time 10. The percentage reduction in the EPV of the survival benefit will be larger than the percentage reduction in the EPV of the death benefit (which is paid early, on average, and is therefore relatively less affected by a change in the interest rate).

Comments: For Part i), the most common error was not calculating the correct discount rate or calculating the net premium instead of the gross premium. For Part ii), many candidates did not really address the question answered and instead commented on expenses or joint life versus single life as opposed to the difference between term and endowment.

Question 6 Model Solutions

(a)

(i) $${}_{7.5}E_{57.5:57.5} = v^{7.5}\ ({}_{7.5}p_{57.5})^2 = v^{7.5}\left(\frac{l_{65}}{l_{57.5}}\right)^2$$

$$= 1.05^{-7.5}\ \left(\frac{94{,}579.7}{(0.5)(97{,}435.2 + 97{,}195.6)}\right)^2 = 0.6551081$$

(ii) $$a^w_{57.5} = v\ \frac{l_{58.5}}{l_{57.5}}\ a^w_{58.5} = \frac{1}{1.05}\ \frac{(0.5)(97{,}195.6+96{,}929.6)}{(0.5)(97{,}435.2+97{,}195.6)}\ (10.5804) = 10.050395$$

Alternatively,

$$\begin{aligned}
a^w_{57.5} &= {}_{7.5}E_{57.5:57.5}\ \ddot{a}^{(12)}_{\overline{65:65}} + {}_{7.5}E_{57.5}\ (1 - {}_{7.5}p_{57.5})\ddot{a}^{(12)}_{65} \\
&= {}_{7.5}E_{57.5}\ \ddot{a}^{(12)}_{65} + {}_{7.5}E_{57.5:57.5}\left(\ddot{a}^{(12)}_{65} - \ddot{a}^{(12)}_{65:65}\right) \\
&= (0.674057)(13.0807) + (0.6551081)(13.0870 - 11.2158) \\
&= 8.8213838 + 1.0258383 = 10.04722
\end{aligned}$$

where

$${}_{7.5}E_{57.5} = 1.05^{-7.5}\ \left(\frac{94{,}579.7}{(0.5)(97{,}435.2 + 97{,}195.6)}\right) = 0.674057$$

Comments: Candidates generally did well on part i. Candidates struggled with part ii. Common errors were misapplied recursive annuity formula and using an annuity due instead of an annuity immediate. The annuity is deferred to age 65 so there is payment at t=0.

(b)

$$AL = (0.018)(35)(100{,}000)\left\{\frac{w_{57}}{l_{57}} v^{0.5}\, a^{w}_{57.5} + \frac{w_{58}}{l_{57}} v^{1.5}\, a^{w}_{58.5} + \frac{w_{59}}{l_{57}} v^{2.5}\, a^{w}_{59.5}\right\}$$

$$= (63{,}000)\left\{\frac{1976}{99{,}960.2} 1.05^{-0.5}(10.04722) + \frac{1929.9}{99{,}960.2} 1.05^{-1.5}(10.5804) + \frac{1884.3}{99{,}960.2} 1.05^{-2.5}(11.1456)\right\}$$

$$= (63{,}000)\{0.193825 + 0.1898566 + 0.1859744\} = 35{,}888.33$$

Comments: Please note that the AL given was not within 100 of the correct AL. As a result of this error, all papers close to the pass mark were reviewed. As a result, several additional candidates were awarded a pass for this exam.

The Accrued Liability was not well done with common errors being:

i. *In general, if candidates applied first principles to the contingent benefit they did well. Trying to shoehorn a rote shortcut did not work well.*
ii. *Candidates applied mortality decrements from SULT instead of retirement decrements from Standard Service Table.*
iii. *Many candidates missed that there were 3 possible withdrawal ages.*

(c)

$$NC = (0.018)(100{,}000)\left\{(0.5)\ \frac{w_{57}}{l_{57}} v^{0.5} a^{w}_{57.5} + \frac{w_{58}}{l_{57}} v^{1.5} a^{w}_{58.5} + \frac{w_{59}}{l_{57}} v^{2.5} a^{w}_{59.5}\right\}$$

$$= (1{,}800)\{(0.5)(0.193825) + 0.1898566 + 0.1859744\} = 850.94$$

Or

$$NC = vp_{57}AL_1 + EPV(mid - year - exits) - AL_0$$

$$= 24{,}353.85 + 12{,}35.46 - 35{,}888.33 = 850.94$$

$$vp_{57}AL_1 = (36)(0.018)(100{,}000)\left\{\left(\frac{w_{58}}{l_{57}}\right)v^{1.5}a^{w}_{58.5} + \left(\frac{w_{59}}{l_{57}}\right)v^{2.5}a^{w}_{59.5}\right\}$$

$$= (64{,}800)\{0.1898566 + 0.185974\} = 24{,}353.85$$

$$EPV(mid - year - exits) = (35.5)(0.018)(100{,}000)\left(\frac{w_{57}}{l_{57}}\right)v^{0.5}a^{w}_{57.5}$$

$$= (63{,}900)(0.193825) = 12{,}385.46$$

Comments: The Normal Cost was also not done well. Common mistakes were:

i. Many PUC shortcut formula for a TUC plan

ii. TUC shortcut didn't work in this case as benefit contingent on withdrawal

iii. Many didn't calculate mid-year exits for withdrawals at age 57

(d)

Value of settlement pre-divorce:
$(0.018)(35.5)(100{,}000)a^{w}_{57.5} = (63{,}900)(10.04722) = 642{,}017.36$

Value of settlement post-divorce:
${}_{7.5}E_{57.5}\ \ddot{a}^{(12)}_{65}\ X\ +\ {}_{7.5}E_{57.5}\ \ddot{a}^{(12)}_{65}\ \frac{X}{3} = (0.674057)(13.0870)\frac{4X}{3} = 11.761845\ X$

For the two values to be equal, X=54,584.75

Comments: Candidates did very poorly on this part. It appeared that the candidates did not understand the questions.

(e)

(i) Greater **after** the divorce settlement: 0 versus 18,195

(ii) Greater **before** the divorce settlement: 63,900 versus 54,584.75

Comments: It appeared that most candidate just guessed at this part.