Fall 2020 LTAM MC Solutions

Question 1: Answer E

Question 2: Answer C

$$E\left[T^2\right] = \int_0^\infty t^2 \, {}_t p_x \, \mu_{x+t} \, dt$$

Using integration by parts

$$= \left[-t^2 \, {}_t p_x\right]_0^\infty + \int_0^\infty 2t \, {}_t p_x \, dt = 0 - 0 + \int_0^\infty 2t \, {}_t p_x \, dt = \int_0^\infty 2t \, {}_t p_x \, dt$$

Question 3: Answer D

Let D denote the number of deaths.

$$D \sim \text{Bin}\left(128, \, {}_4 q_{[2]+1}\right) \Rightarrow \text{Var}\left[D\right] = (128)\left({}_4 q_{[2]+1}\right)\left(1 - {}_4 q_{[2]+1}\right)$$

$${}_4 q_{[2]+1} = 1 - \frac{l_7}{l_{[2]+1}} = 0.375$$

$$\text{Var}\left[D\right] = (128)(0.375)(1 - 0.375) = 30$$

Question 4: Answer C

Question 5: Answer A

$$q^{(1)} = q'^{(1)}\left(1 - \tfrac{1}{2} q'^{(2)}\right) \Rightarrow q'^{(1)} = \frac{q^{(1)}}{1 - \frac{1}{2} q'^{(2)}} = \frac{0.138}{1 - \frac{1}{2}\left(0.070\right)} = 0.143005$$

$$q^{(2)} = q'^{(2)}\left(1 - \tfrac{1}{2} q'^{(1)}\right) = \left(0.070\right)\left(1 - \left(0.5\right)\left(0.143005\right)\right) = 0.064995$$

$$1000 q^{(2)} = 65$$

Question 6: Answer B

$$r_1 = 50$$

$$\Rightarrow H\left(y_2\right) = \frac{1}{50} + \frac{1}{49} = 0.04041 \Rightarrow S\left(y_2\right) = e^{-0.04041} = 0.9604$$

Question 7: Answer B

$$\begin{aligned}\ddot{a}_{[60]+1} &= 1+v\,p_{[60]+1}+v^2\,{}_2p_{[60]+1}\,\ddot{a}_{63}\\ &= 1+v\left(\frac{95{,}910}{96{,}197}\right)+v^2\left(\frac{95{,}534}{96{,}197}\right)12.886\\ &= 13.330\end{aligned}$$

Question 8: Answer A

$$\begin{aligned}800{,}000\bar{A}^{03}_{55:\overline{20}|} &= 800{,}000\left(\bar{A}^{03}_{55}-{}_{20}p^{00}_{55}\times v^{20}\times\bar{A}^{03}_{75}-{}_{20}p^{01}_{55}\times v^{20}\times\bar{A}^{13}_{75}\right)\\ &= 800{,}000\left(0.1856-(0.7335)(1.05)^{-20}(0.4910)-(0.1361)(1.05)^{-20}(0.7657)\right)\\ &= 8470\end{aligned}$$

Question 9: Answer E

$$\bar{A}^{02}_{50}=\int_0^\infty {}_tp^{00}_{50}\,\mu^{02}_{50+t}\,e^{-\delta t}\,dt \quad \text{where} \quad {}_tp^{00}_x={}_tp_x \quad \text{from SULT}$$

$$\Rightarrow \bar{A}^{02}_{50}=0.00022\int_0^\infty {}_tp_{50}\,e^{-\delta t}\,dt=0.00022\,\bar{a}_{50}=0.00022(16.520)=0.0036344$$

$$\Rightarrow 200{,}000\bar{A}^{02}_{50}=(200{,}000)(0.0036344)=727$$

Question 10: Answer E

EPV Premiums: $0.5P\left(\ddot{a}_{50:\overline{10}|}+\ddot{a}_{60:\overline{10}|}\right)=0.5P\left(8.0550+7.9555\right)=8.00525P$

EPV Benefits: $1{,}000{,}000\left(A^1_{50:\overline{10}|}+A^1_{60:\overline{10}|}-A^{\overline{1}}_{\overline{50:60}:\overline{10}|}\right)$

$=1{,}000{,}000\left(0.01461+0.04252-0.05636\right)=770$

$$A^1_{50:\overline{10}|}=A_{50:\overline{10}|}-{}_{10}E_{50}=0.61643-0.60182=0.01461$$

$$A^1_{60:\overline{10}|}=A_{60:\overline{10}|}-{}_{10}E_{60}=0.62116-0.57864=0.04252$$

$$A^{\overline{1}}_{\overline{50:60}:\overline{10}|}=A_{50:60}-{}_{10}p_{50:60}v^{10}A_{60:70}=0.32048-\left(\frac{91{,}082.4}{98{,}576.4}\right)(1.05)^{-10}(0.46562)=0.05636$$

$$\Rightarrow P=\frac{770}{8.00525}=96.19$$

Question 11: Answer D

$$100{,}000A^{1}_{30:\overline{35}|} = 12{,}000 \Rightarrow A^{1}_{30:\overline{35}|} = 0.12$$

$$11{,}865 = 100{,}000A^{1}_{30:\overline{35}|} - 90{,}000\,{}_{\frac{1}{4}}q_{30}v \Rightarrow {}_{\frac{1}{4}}q_{30} = \frac{100{,}000(0.12) - 11{,}865}{90{,}000}1.03 = 0.001545$$

$${}_{\frac{1}{4}}q_{30} = \left(\frac{1}{4}\right)q_{30} = 0.001545 \Rightarrow q_{30} = 0.00618$$

Question 12: Answer D

Actuarial Present Value of Premiums = $G\ddot{a}_{65:\overline{10}|}$

Actuarial Present Value of Benefits = $100{,}000A^{1}_{65:\overline{10}|} + 10 \cdot G \cdot {}_{10}E_{65}$

Actuarial Present Value of Benefits = $2000 + 0.475G + 0.025G\ddot{a}_{65:\overline{10}|} + 500A_{65:\overline{10}|}$

$$\Rightarrow G = \frac{100{,}000A^{1}_{65:\overline{10}|} + 2000 + 500A_{65:\overline{10}|}}{0.975\ddot{a}_{65:\overline{10}|} - 0.475 - 10\,{}_{10}E_{65}}$$

$$= \frac{100{,}000(0.62650 - 0.55305) + 2000 + 500(0.62650)}{0.975(7.8435) - 0.475 - 10\,(0.55305)} = 5882.32$$

Question 13: Answer C

$$P = \frac{12{,}000\bar{a}^{01}_{55} + 100{,}000\bar{A}^{02}_{55}}{\bar{a}^{00}_{55}} = \frac{(12{,}000)(2.3057) + (100{,}000)(0.39366)}{10.1228} = 6622.12$$

Question 14: Answer C

$$G = \frac{250{,}000A_{35} + 100\ddot{a}_{35} + 400}{0.92\ddot{a}_{35} - 0.22}$$

$$= \frac{(250{,}000)(0.09653) + (100)(18.9728) + 400}{(0.92)(18.9728) - 0.22} = 1533.50$$

$${}_{3}V = 250{,}000A_{38} + 100\ddot{a}_{38} - 1533.50 \times 0.92 \times \ddot{a}_{38}$$

$$= (250{,}000)(0.11059) + (100)(18.6777) - (1533.50)(0.92)(18.6777) = 3164.40$$

Question 15: Answer A

$$\begin{aligned}\text{Profit} &= \left({}_2V + P - E\right)\left(1.0075\right) - {}_{\frac{1}{12}}q_{60} \times 75{,}000 - {}_{\frac{1}{12}}p_{60}\left(1 - withdrawals\right){}_{2\frac{1}{12}}V \\ &= \left(2430 + 100 - 5\right)\left(1.0075\right) - \left(0.000283\right)\left(75{,}000\right) - \left(1 - 0.000283\right)\left(0.9975\right)\left(2520\right) = 9.72\end{aligned}$$

Question 16: Answer D

$$PM = 0.087 = \frac{-162 + 60v_r + 175p_{80}v_r^2}{100\left(1 + v_r\ p_{80}\right)}$$

$$\Rightarrow 148.75v_r^2 + 52.605v_r - 170.7 = 0$$

$$\Rightarrow v_r = \frac{-52.605 + \sqrt{52.605^2 + 4 \times 148.75 \times 170.7}}{2 \times 148.75} = 0.90892$$

$$\Rightarrow r = \frac{1}{v_r} - 1 = 10.0\%$$

Question 17: Answer E

$$NC = 108{,}650 \times 0.018 \times {}_{15}E_{50} \times \ddot{a}_{65} = 12{,}230$$

Question 18: Answer A

$${}_{9.5}V = \frac{({}_9V + (0.95)P_9)(1+i)^{0.5} - (100{,}000)(v^{0.5})({}_{0.5}q_{84})}{1 - {}_{0.5}\ q_{84}}$$

$$= \frac{(17{,}138 + (0.95)(7200))(1.05)^{0.5} - (100{,}000)(1.05)^{0.5}(0.5)(0.051493)}{1 - (0.5)(0.051493)} = 22{,}640$$

Question 19: Answer B

$$\text{AVTHB}= B(64,0)\left(1.03^{0.5}\times 1.04^{0.5}\times v^{0.5}\times\frac{r_{64}}{l_{64}}\times\ddot{a}_{64.5|i^*}+1.03\times 1.04\times v\times\frac{r_{65}}{l_{64}}\times\ddot{a}_{65|i^*}\right)$$

$$=6396\left(v_{i^*}^{0.5}\times\frac{r_{64}}{l_{64}}\times\ddot{a}_{64.5|i^*}+v_{i^*}\times\frac{r_{65}}{l_{64}}\times\ddot{a}_{65|i^*}\right)$$

$$=6396\left(\left[\frac{(1.04)(1.03)}{1.06}\right]^{0.5}\left[\frac{4{,}061.0}{42{,}805.0}\right](27.2753)+\left[\frac{(1.04)(1.03)}{1.06}\right]\left[\frac{38{,}488.3}{42{,}805.0}\right](26.7083)\right)$$

$$=6396\left(2.6013+24.2686\right)=171{,}860$$

Question 20: Answer D

$$\frac{d}{dt}{}_tV^g=\delta\times 46{,}531+0.95\left(2500\right)-0.031313\left(101{,}000-46{,}531\right)=2531$$