LTAM Exam
Spring 2022
Solutions

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Solutions to Multiple Choice Questions

1. **Answer: B**

2. **Answer: A**

Let N denote the number of lives in the group.

$$N \, {}_2p_{[50]} = N\left(\frac{l_{[50]+2}}{l_{[50]}}\right) = N\left(\frac{100,000}{110,000}\right) = 200 \Rightarrow N = 220$$

$$N \, {}_2p_{[48]+2} = N\left(\frac{l_{52}}{l_{[48]+2}}\right) = N\left(\frac{102,000}{120,000}\right) = 220(0.85) = 187$$

3. **Answer: E**

$$\mu_x = -\frac{d}{dx}\log S_0(x) = -\frac{d}{dx} 0.5\log(1-0.01x) = 0.005\frac{1}{1-0.01x}$$

$$\Rightarrow \mu_{25} = 0.00667 \Rightarrow 1000\mu_{25} = 6.7$$

4. **Answer: D**

$$\hat{S}(7) = \frac{49}{50} \times \frac{69}{70} = 0.966$$

5. **Answer: D**

$$\int_0^\infty {}_tp_x^{\overline{00}} \cdot \mu_{x+t}^{01} \cdot \underbrace{{}_1p_{x+t}^{12}} \cdot dt$$

probability   This   probability
that a life in   transitions   that a life
state 0 to   life from   in state 1
start has   0 to 1   at age x+t
stayed in 0   at age   will be in
until   x+t   state 2
transition   after
                  one year

6. **Answer: E**

$$q_{65}^{(1)} + q_{65}^{(2)} = 1 - \frac{l_{66}^{(\tau)}}{l_{65}^{(\tau)}} = 1 - \frac{9600}{10,000} = 0.04 \Longrightarrow q_{65}^{(2)} = 0.04 - 0.03 = 0.01$$

$$\Longrightarrow q_{67}^{(1)} = 0.05$$

$$q_{67}^{(1)} + q_{67}^{(1)} = 1 - \frac{l_{68}^{(\tau)}}{l_{67}^{(\tau)}} = 1 - \frac{8436}{9120} = 0.075$$

$$\Rightarrow q_{67}^{(2)} = 0.075 - 0.05 = 0.025$$

$$\Rightarrow d_{67}^{(2)} = 0.025 \times 9120 = 228$$

7. Answer: D

$$A_{50}^{(2)} = {}_{0.5}q_{50}v^{0.5} + {}_{0.5}p_{50}\,v^{0.5}A_{50.5}^{(2)}$$

$$v^{0.5} = \frac{1}{1.03}$$

$${}_{0.5}q_{50} = 0.5 \times (1 - 0.99850) = 0.00075; \quad {}_{0.5}p_{50} = 0.99925$$

$$\Rightarrow A_{50.5}^{(2)} = \frac{0.1772 - 0.00075v^{0.5}}{0.99925v^{0.5}} = 0.18190$$

8. Answer: B

$$p_{70}^{HH} = 0.86; \quad {}_{2}p_{70}^{HH} = 0.86 \times 0.84 + 0.12 \times 0.08 = 0.732$$
$$p_{70}^{HS} = 0.12; \quad {}_{2}p_{70}^{HS} = 0.12 \times 0.66 + 0.86 \times 0.13 = 0.191$$
$$p_{70}^{HD} = 0.02; \quad {}_{2}p_{70}^{HD} = 0.02 + 0.86 \times 0.03 + 0.12 \times 0.26 = 0.077$$
$$v = (1/1.02)$$

$$EPV = 12,000\left(0.86v + 0.732v^2\right) + 30,000(0.12v + 0.191v^2) + 40,000(0.02v + (0.077 - 0.02)v^2)$$
$$= 18,560.55 + 9036.91 + 2975.78 = 30,573.24$$

9. Answer: A

$$P = 100,000(A_{50:\overline{10|}}^{1} + A_{60:\overline{10|}}^{1} - A_{\overline{50:60:10|}}^{\quad 1})$$

$$A_{50:\overline{10|}}^{1} = A_{50:\overline{10|}} - {}_{10}E_{50} = 0.61643 - 0.60182 = 0.01461$$
$$A_{60:\overline{10|}}^{1} = A_{60:\overline{10|}} - {}_{10}E_{60} = 0.62116 - 0.57864 = 0.04252$$
$$A_{\overline{50:60:10|}}^{\quad 1} = \left(1 - d\ddot{a}_{50:60:\overline{10|}} - {}_{10}E_{50:60}\right) = 1 - d(7.9044) - (1.05)^{10}(0.60182)(0.57864) = 0.05636$$

$$\Rightarrow P = 100,000(0.01461 + 0.04252 - 0.05636) = 77.0$$

10. Answer: C

$$EPV = 10,000 \int_0^{10} {}_tp_{xy}^{00} \left(\mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{02} \right) e^{-\delta t} \, dt$$

$$= 10,000 \int_0^{10} e^{-(0.20t)} (0.16) e^{-0.0433t} \, dt = 10,000(0.16) \left(\frac{1-e^{-2.433}}{0.2433} \right)$$

$$= 5999.0$$

11. Answer: B

$$E[L] = 200 \left(150,000 A_{40} - 1140 \ddot{a}_{40} \right) = 200 \left([150,000][0.12106] - [1140][18.4578] \right) = -576,578$$

$$SD[L] = \sqrt{200} \left(\left(150,000 + \frac{1140}{d} \right) \left(\sqrt{{}^2A_{40} - A_{40}^2} \right) \right)$$

$$= \sqrt{200} \left(\left(150,000 + \frac{1140}{0.05/1.05} \right) \left(\sqrt{0.02347 - (0.12106)^2} \right) \right) = \sqrt{200} (16,330) = 230,947$$

$$95\%\text{ile} = -576,578 + 1.645 \times 230,947 = -196,670$$

12. Answer: B

$$P\ddot{a}_{55:\overline{10}|} = 12 \times 5000 \times {}_{10}E_{55} \times \ddot{a}_{65}^{(12)}$$

$$\ddot{a}_{65}^{(12)} = 13.5498 - 11/24 = 13.0915$$

$$\Rightarrow P = \frac{(12)(5000)(0.59342)(13.0915)}{8.0192} = 58,126$$

13. Answer: A

$$12P\ddot{a}_{55:\overline{10}|}^{00(12)} = 12(5000)\ddot{a}_{55:\overline{10}|}^{01(12)}$$

$$\ddot{a}_{55:\overline{10}|}^{01(12)} = \ddot{a}_{55}^{01(12)} - v^{10}\,_{10}p_{55}^{00}\,\ddot{a}_{65}^{01(12)} - v^{10}\,_{10}p_{55}^{01}\,\ddot{a}_{65}^{11(12)}$$

$$\ddot{a}_{55}^{01(12)} = \bar{a}_{55}^{01} = 2.3057; \quad \ddot{a}_{65}^{01(12)} = \bar{a}_{65}^{01} = 2.8851$$

$$\ddot{a}_{65}^{11(12)} = \bar{a}_{65}^{11} + \frac{1}{24} = 8.8540$$

$$\Rightarrow \ddot{a}_{55:\overline{10}|}^{01(12)} = 2.3057 - (1.05)^{-10}(0.74091)(2.8851) - (1.05)^{-10}(0.11682)(8.8540) = 0.3584$$

$$\Rightarrow P = \frac{(12)(5000)(0.3584)}{(12)(7.1253)} = 251.5$$

14. Answer: D

$$P\ddot{a}_{60:70} = 100,000 A_{60:70} + 2P\left(A_{60} + A_{70} - A_{60:70}\right)$$

$$\Rightarrow P = \frac{100,000(0.46562)}{11.2220 - 2\left(0.29028 + 0.42818 - 0.46562\right)} = 4344.96$$

15. Answer: E

$$_{20}V = 125,000 A_{55} + 125,000 A_{55:\overline{10}|}^{1} + 25\ddot{a}_{55} - 0.95(1000)\ddot{a}_{55:\overline{10}|}$$

$$A_{55:\overline{10}|}^{1} = A_{55:\overline{10}|} - {}_{10}E_{55} = 0.02471$$

$$\Rightarrow {}_{20}V = 125,000(0.23524 + 0.02471) + 25(16.0599) - (0.95)(1000)(8.0192)$$
$$= 25,277.0$$

16. **Answer: C**

$$P_{x+1}^{FPT} = 100,000\left(\frac{A_{46}}{\ddot{a}_{46}}\right) = 100,000\left(\frac{1 - d\ddot{a}_{46}}{\ddot{a}_{46}}\right)$$

$$\ddot{a}_{46} = \frac{\ddot{a}_{45} - 1}{vp_{45}} = \frac{20.45115 - 1}{(1.04)^{-1}(1 - 0.00077)} = 20.24478$$

$$P_{x+1}^{FPT} = 100,000\left(\frac{1 - (0.04/1.04)(20.24478)}{20.24478}\right) = 1093.39$$

$$\left(_{20}V + P\right)(1+i) = 100,000 \times q_{65} + p_{65} \times _{21}V$$

$$\Rightarrow _{21}V = \frac{(26,526.57 + 1093.39)(1.04) - 591.0}{1 - 0.00591} = 28,301$$

17. **Answer: A**

$$\frac{d}{dt}_tV^{(0)} = \delta _tV^{(0)} + P - E - \mu_{x+t}^{01}\left(0.6 _tV^{(0)} + 1,000 - _tV^{(0)}\right) - \mu_{x+t}^{02}\left(501,000 - _tV^{(0)}\right)$$

$$= 0.05(200,000) + 6,000 - 150 - 0.03\left(1,000 - 0.4(200,000)\right) - 0.018\left(501,000 - 200,000\right)$$

$$= 12,802.0 \text{ at } t = 25$$

18. **Answer: E**

The DPP is t such that $NPV(t) \geq 0$, with $NPV(t-1) < 0$, $t \in \mathbb{Z}^+$

$$NPV(t) \geq 0 \Leftrightarrow -220 + 31v + 45a_{\overline{t}|} \geq 0 \Leftrightarrow a_{\overline{t}|} \geq 4.3148$$

$$\Leftrightarrow \frac{1 - v_{20\%}^t}{0.2} \geq 4.3148 \Leftrightarrow v_{20\%}^t \leq 0.13704$$

$$\Rightarrow t \geq 10.9$$

$$\Rightarrow t = 11 \text{ as } t \text{ is an integer.}$$

19. Answer: C

The service at the valuation date is 15 years, so

$$NC = AL\left(\left(\frac{16}{15}\right)(1.025) - 1\right) = 14,155$$

20. Answer: D

Let $j = 0.0269$ and let $c = 1.025$:

$$EPV = 4500 \times (1+j)^{29} \times v^{29} \times {}_{29}p_{35} \times \left(1 + p_{64}\, v(1+j)c\right)$$

$$= 4500(1.0269)^{29}(1.05)^{-29}\left(\frac{95,082.5}{99,556.7}\right)(1 + (1 - 0.005288)(1.05)^{-1}(1.0269)(1.025))$$

$$= 4503$$

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Solutions to Written Answer Questions

Question 1 Model Solution

(a)

$$P(0.95\ddot{a}_{50:\overline{30}|} - 0.1) = 401,000 A_{50:\overline{30}|} + 500 + 40\ddot{a}_{50:\overline{30}|}$$

$$\ddot{a}_{50:\overline{30}|} = \ddot{a}_{50} - {}_{30}E_{50}\ \ddot{a}_{80} = 17.0245 - 0.17758(8.5484) = 15.5065$$

$$A_{50:\overline{30}|} = 1 - d\ddot{a}_{50:\overline{30}|} = 0.26160$$

or

$$A_{50:\overline{30}|} = A_{50} - {}_{30}E_{50} \cdot A_{80} + {}_{30}E_{50}$$

$$= 0.18931 - (0.34824)(0.50994)(0.59293) + (0.34824)(0.50994)$$

$$= 0.26160$$

$$\Rightarrow P = 7246.30$$

Comments: Most of the candidates received full credit on this part. The most common error was treating the insurance as a term insurance instead of an endowment insurance.

(b)

(i) $L_0^g = 401,000 v^{\min(K_{50},30)} + 500 + 0.1(7500) - \left(0.95(7500) - 40\right)\ddot{a}_{\overline{\min(K_{50},30)}|}$

$$= 401,000 v^{\min(K_{50},30)} + 1,250 - 7,085\left(\frac{1 - v^{\min(K_{50},30)}}{d}\right)$$

$$= 549,785 v^{\min(K_{50},30)} - 147,535$$

$$\Rightarrow A = 549,785;\quad B = -147,535$$

(ii) $E\left[L_0^g\right] = 549,785 A_{50:\overline{30}|} - 147,535 = -3711.2$

(iii) $V\left[L_0^g\right] = \left(401,000 + \dfrac{7085}{d}\right)^2 \left({}^2A_{50:\overline{30}|} - A_{50:\overline{30}|}^2\right)$

or straight to

$$= 549,785^2 \left({}^2A_{50:\overline{30}|} - A_{50:\overline{30}|}^2\right)$$

$${}^2A_{50:\overline{30}|} = {}^2A_{50} - v^{30}\,{}_{30}E_{50}\,{}^2A_{80} + v^{30}\,{}_{30}E_{50} = 0.07650$$

$$\Rightarrow SD\left[L_0^g\right] = 549,785(0.00806)^{0.5} = 49,375$$

(c)

Let X denote the monthly payment.

$$400,000 = 12X\left(\ddot{a}_{\overline{10|}}^{(12)} + {}_{10}E_{80}\,\ddot{a}_{90}^{(12)}\right)$$

$$\ddot{a}_{\overline{10|}}^{(12)} = 7.9293; \qquad \ddot{a}_{90}^{(12)} = 5.1835 - 11/24 = 4.72517$$

$$\Rightarrow X = \frac{400,000}{12[7.9293 + (0.33952)(4.72517)]} = \frac{400,000}{114.40308} = 3496.41$$

(d)

Anti-selection risk – if the lives selecting this option live much longer than the average, the option will be more costly than anticipated.

Longevity risk – if all lives live much longer than expected, the option will be more costly than expected (though there may be offsets from death benefits).

Interest rate risk - the insurer is guaranteeing an annuitization rate far into the future. If interest rates are lower at that time, there will be a loss. (The maturity date is too far ahead to hedge the interest rate risk)

Expense Risk – No expenses are included to account for the cost of making these payments.

(e)

- Does Jenna believe her future lifetime will be greater than average? If so, she may choose the annuity
- What is the market value of the annuity at the maturity date? If it is lower (eg because interest rates have increased) then Jenna should take the lump sum and buy the annuity separately.
- The lump sum offers greater liquidity benefits – how important are they to Jenna?
- The lump sum brings greater dissipation risk – does Jenna need a steady, guaranteed income stream? If so, she may choose to annuitize.
- How will taxation impact the proceeds under the two options?
- How important is having money to leave to her survivors if she dies? Taking the lump sum option gives the opportunity for a large bequest if she dies soon after age 80. If she takes the annuity option, she will have the balance of the guarantee for bequest, if she dies before age 90.

Question 2 Model Solution

(a)

(i) $\bar{A}^{02}_{60:\overline{10}|} = \bar{A}^{02}_{60} - {}_{10}p^{00}_{60} \; v^{10} \; \bar{A}^{02}_{70} - {}_{10}p^{01}_{60} \; v^{10} \; \bar{A}^{12}_{70} =$

$0.39077 - (0.75055)(1.05)^{-10}(0.54335) - (0.13135)(1.05)^{-10}(0.62237)$

$= 0.09022$

(ii) ${}_{10}V^{(0)} = 500,000 \; \bar{A}^{02}_{60:\overline{10}|} + 100,000\bar{A}^{01}_{60:\overline{10}|} - 4850\bar{a}^{00}_{60:\overline{10}|} = 21,850.4$

$\bar{a}^{00}_{60:\overline{10}|} = \bar{a}^{00}_{60} - {}_{10}p^{00}_{60} \; v^{10} \; \bar{a}^{00}_{70} = 7.1461$

$\bar{A}^{01}_{60:\overline{10}|} = \bar{A}^{01}_{60} - {}_{10}p^{00}_{60} \; v^{10} \bar{A}^{01}_{70} = 0.11397$

(iii) ${}_{10}V^{(1)} = 500,000\bar{A}^{12}_{60:\overline{10}|} = 95,700$

$\bar{A}^{12}_{60:\overline{10}|} = \bar{A}^{12}_{60} - {}_{10} \; p^{11}_{60}v^{10} \; A^{12}_{70} = 0.47904 - 90.75283)(1.05)^{-10}(0.62237) = 0.19140$

Comments: Most candidates were able to get the majority of the points on Part a.

(b)

(i) $\dfrac{d}{dt} {}_{t}V^{(0)} = \delta \, {}_{t}V^{(0)} + P - \mu^{01}_{x+t}\left(100,000 + {}_{t}V^{(1)} - {}_{t}V^{(0)}\right) - \mu^{02}_{x+t}\left(500,000 - {}_{t}V^{(0)}\right)$

$[\ln(1.05)](21,850) + 4850 - (0.00818)(100,000 + 95,700 - 21,850) - (0.00724)(500,000 - 21,850)$

$= 1032.2$ at $t = 10$

(ii) $\dfrac{d}{dt} {}_{t}V^{(1)} = \delta \, {}_{t}V^{(1)} - \mu^{12}_{x+t}\left(500,000 - {}_{t}V^{(1)}\right) = [\ln(1.05)](95,700) - (0.01811)(500,000 - 97,500)$

$= -2652.8$ at $t = 10$

(iii) In State 0, the cash flows in are interest and premiums, and the cash flows out are claims and reserves on transition to State 1, and claims on transition to State 2.

In the early years of the term insurance the State 0 reserve will gradually increase, as premiums exceed expected outgo.   In later years the reserve will decrease, as the greater number of claims outweighs the premium and investment income.   At time 10, the reserve is still in the increasing phase.

If a policy is in state 1 it is effectively a single premium term insurance. There is a little interest income, but no premium income.   The outgo on claims, on average, depletes the reserve, which will decline to 0 just before the end of the contract.

Comments: Candidates struggled considerably with this part. Many candidates skipped part iii.

(c)

$$100,000 \int_0^{20} {}_t p_x^{00} \; \mu_{x+t}^{01} \; {}_{0.5}p_{x+t}^{11} \; v^{t+0.5} \; dt$$

Comments: Candidates who attempted this part received the majority of the points.

(d)

(i) Under the Markov property, for a policy in State 1 at time t, the probability of moving from State 1 to State 2 in any future time interval is independent of the history of the state process before time t.

This means, that (for example) the probability of a life dying in the next 6 months for a policyholder currently in State 1, is the same if they were diagnosed, say, in the last week as if they were diagnosed several years ago.

(ii) This is inconsistent with the spike in mortality observed after diagnosis, as this implies a higher mortality rate for lives recently arrived in State 2, compared with lives who have been in State 2 for a long time.

Comments: Many candidates skipped this part.

Question 3 Model Solution

General Comment: Even if you don't know the CBD model, you can do a lot of this from the basic definitions given in the question.

(a)

Stochastic longevity models help to capture and understand the increasing uncertainty in mortality for survival models. They can capture randomness and systematic trends related to age effects, cohort effects and year effects.

(b)

The "year effects" refers to conditions that affect mortality at a range of ages over one or more calendar years. Examples of this phenomenon are climate effects that may cause a rise in mortality for old ages in a year with extremely hot temperatures, and the epidemiological effects such as pandemics that temporarily increase mortality across a range of ages.

Comments: Many candidates conflated the idea of stochastic mortality models with any mortality model that includes age and year effects. Also some candidates confused the year effect with the cohort effect.

(c)

(i) $E\left[lq(70,2022)\right] = E\left[K^{(1)}_{2022} - 3K^{(2)}_{2022}\right]$

$= E\left[-4.50 - 0.01 + 0.05Z^{(1)}_{2022} - 3\left(0.02 + 0.015Z^{(2)}_{2022}\right)\right] = -4.57$

(ii) $V\left[lq(70,2022)\right] = V\left[-4.50 - 0.01 + 0.05Z^{(1)}_{2022} - 3\left(0.02 + 0.015Z^{(2)}_{2022}\right)\right]$

$= V\left[0.05Z^{(1)}_{2022} - 0.045Z^{(2)}_{2022}\right]$

$= E\left[\left(0.05Z^{(1)}_{2022} - 0.045Z^{(2)}_{2022}\right)^2\right] \quad \left(\text{as } E\left[Z^{(1)}_{2022}\right] = E\left[Z^{(2)}_{2022}\right] = 0\right)$

$= E\left[0.0025\left(Z^{(1)}_{2022}\right)^2 + 0.002025\left(Z^{(2)}_{2022}\right)^2 - 2(0.00225)Z^{(1)}_{2022}Z^{(2)}_{2022}\right]$

$= 0.0025 + 0.002025 - 0.0045(0.15) \quad \left(\text{as } E\left[\left(Z^{(1)}_{2022}\right)^2\right] = E\left[\left(Z^{(2)}_{2022}\right)^2\right] = 1\right)$

$= 0.00385 = 0.06205^2$

Comments: Many candidates had errors in the covariance term (using -2 as the coefficient, forgetting the 2, or forgetting the covariance entirely).

(d)

(i) $\dfrac{d}{dq}\left(\log\dfrac{q}{1-q}\right) = \dfrac{d}{dq}\left(\log q - \log(1-q)\right) = \dfrac{1}{q} + \dfrac{1}{1-q} > 0$ for $0 < q < 1$

Alternately

$$\dfrac{d}{dq}\left(\log\dfrac{q}{1-q}\right) = \left(\dfrac{1-q}{q}\right)\dfrac{(1-q)+q}{(1-q)^2} = \dfrac{1}{q(1-q)} = \dfrac{1}{q} + \dfrac{1}{1-q}$$

(ii) As lq is a linear function of normal random variables, it is also normally distributed.

That is, $lq \sim N(-4.57,\ 0.062^2)$

if $q = 0.01$ *then*
$$lq = ln\left(\dfrac{0.01}{0.99}\right) = -4.5951$$
$$P(q \le 0.01) = P(lq \le -4.5951) \ as \ lq \ increases \ with \ q$$

$Pr(lq \le -4.5951) = \Phi\left(\dfrac{-4.5951-(-4.57)}{0.062}\right) = \Phi(-0.405) \approx 1 - 0.6591 = 0.3409.$

Comments: Many candidates tried to use Normal distribution to calculate a lognormal probability.

Question 4 Model Solution

(a)

The future lifetimes are not independent. The mortality of each life is different when their partner is alive than it is when their partner is dead. This shows that the future lifetime of each partner impacts the future lifetime of the other.   OR: A necessary condition for independence is that $\mu_y^{01} = \mu_y^{23}$ and $\mu_x^{02} = \mu_x^{13}$. As this is not the case, the future lifetimes are not independent.

Comments: Most candidates who correctly identified dependence gave good reasoning. Many candidates failed to recognize dependence, assuming that since they couldn't move directly from State 0 to State 3 meant independence.

(b)

$$_tp_{xy}^{00} = \exp\left(-\int_0^t \mu_{x+r:y+r}^{01} + \mu_{x+r:y+r}^{02} \ dr\right) = \exp\left(-\int_0^t \mu_{y+r}^s + \mu_{x+r}^s - 0.005 \ dr\right)$$

$$= e^{0.005t} \ _tp_x^s \ _tp_y^s \quad ==> \ \lambda = 0.005$$

Comments: This is a show that question, where candidates have the answer. They need to be very precise about their derivation to prove they aren't fudging it. Overall candidates did well on this part. However, many candidates assumed constant mortality, which is incorrect.

(c)

$$\bar{a}_{x:y}^{00} = \int_0^\infty \ _tp_{x:y}^{00} \ e^{-\delta t} \ dt = \int_0^\infty \ _tp_x^s \ _tp_y^s \ e^{\lambda t} e^{-\delta t} \ dt = \bar{a}_{x:y}^s \Big|_{\delta^*} \quad \text{where} \ \delta^* = \delta - \lambda = 0.035$$

$$\Rightarrow \bar{a}_{40:50}^{00} = 19.3199$$

Comments: The point of this part is to get candidates to work out the correct interest rate to use. For full credit, they need to justify their choice. Many candidates overcomplicated the question, spending time integrating, instead of looking up on the table (this was especially true for candidates who assumed constant force of mortality).

(d)

(i) $\overline{A}_{x:y}^{01} + \overline{A}_{x:y}^{02} = \int\limits_{0}^{\infty} {}_t p_{x:y}^{00}\left(\mu_{y+t}^{01} + \mu_{x+t}^{02}\right)e^{-\delta t}\,dt$

(ii) $\overline{A}_{x:y}^{01} + \overline{A}_{x:y}^{02} = \int\limits_{0}^{\infty} {}_t p_x^{s}\,{}_t p_y^{s}\left(\mu_{y+t}^{s} + \mu_{x+t}^{s} - 0.005\right)e^{-\delta^* t}\,dt$

$$= \overline{A}_{x:y\,|\delta^*}^{s} - 0.005\,\overline{a}_{x:y\,|\delta^*}^{s} = 1 - \delta^*\overline{a}_{x:y\,|\delta^*}^{s} - 0.005\,\overline{a}_{x:y\,|\delta^*}^{s}$$

$$= 1 - \delta\overline{a}_{x:y\,|\delta^*}^{s} = 1 - (0.04)(19.3199) = 0.227204$$

Comments: Most candidates either skipped this part or utilized a constant force of mortality.

(e) $P = \dfrac{500,000\left(\overline{A}_{x:y}^{01} + \overline{A}_{x:y}^{02}\right)}{\overline{a}_{x:y}^{00}} = \dfrac{(500,000)(0.227204)}{19.3199} = 5,880.1$

(f)

$$X = \frac{500,000}{\overline{a}_{50}^{11}} = \frac{500,000}{\overline{a}_{x:y\,|\delta^{**}=0.0425}^{s}} = \frac{500,000}{17.9917} = 27,791$$

Comments: Parts (e) and (f) were very basic, relying on answers from earlier parts. However, many candidates who struggled with part d skipped these parts, even though part d had the numeric answer.

Question 5 Model Solution

(a)

    (i)   The EPV at age 63.5 is

$$EPV = 12.5 \times 0.016 \times FAS_{63.5} \times (1 - 18 \times 0.004) \times \ddot{a}^{(12)}_{63.5}$$
$$\text{where } FAS_{63.5} = 0.5 \times 72{,}100 + 0.5 \times 70{,}000 = 71{,}050$$
$$EPV = 13{,}186.88 \times 13.5139 = 178{,}206.18$$

    (ii)   The EPV at 1 Jan 2022 of the mid-year exits is

$$\frac{r_{63}}{l_{63}} v^{0.5}(178{,}206.18) = \frac{4{,}515.2}{47{,}579.3} 1.05^{-0.5}(178{,}206.18)$$
$$= 16{,}503.92 \text{ (or 16,503.90 if using 178,206)}$$

(b)

    (i)   The projected actuarial liability (AL) at 1/1/2023 is the value at 1/1/2023 of the exit benefits for ages 64.5 and 65, based on 13 years of past service.

              The AL at 1/1/2022 is the value at 1/1/2022 of the exit benefits for ages 63.5, 64.5 and 65, based on 12 years of past service. This gives us

$$AL = \frac{12}{12.5} \times 16{,}503.92 + \frac{12}{13} \times v \times \frac{l_{64}}{l_{63}} \times 191{,}309$$

$$= \frac{12}{12.5} \times 16{,}503.92 + \frac{12}{13} \times 1.05^{-1} \times \frac{42{,}805.0}{47{,}579.3} \times 191{,}309$$

$$= 15{,}843.76 + 151{,}307.50 = 167{,}151.26$$

    (ii)   The normal cost is the value at 1/1/2022 of the mid-year exits plus the value at 1/1/2022 of the projected 1/1/2023 AL minus the 1/1/2022 AL. That is

$$NC = 16{,}503.92 + 191{,}309 \times \frac{l_{64}}{l_{63}} \times v - 167{,}151.26 = 13{,}269.11$$

              OR

              There is another way of calculating the normal cost, which funds a half-year for the upcoming midyear exits and the full year for the rest.

$$AL = \frac{0.5}{12.5} \times 16{,}503.92 + \frac{1}{13} \times v \times \frac{l_{64}}{l_{63}} \times 191{,}309$$

$$= \frac{0.5}{12.5} \times 16{,}503.92 + \frac{1}{13} \times 1.05^{-1} \times \frac{42{,}805.0}{47{,}579.3} \times 191{,}309$$

$$= 660.16 + 12{,}608.96 = 13{,}269.11$$

Comments: For both parts (a) and (b), read carefully. When it comes to pension plans, timing is everything! Not only is the exact time when the person retires matters, but also when the benefit is valued. Projected unit credit projects salaries, not service, from the valuation date. Be careful to make use of information provided, which you need not reproduce unless explicitly asked to.

(c)

    (i)   The AL using TUC will be <u>smaller</u> than the PUC AL.

- The TUC AL does not project the FAS to retirement, using the current FAS instead. As salaries are increasing, this means that the TUC AL is always less than the PUC AL, except at the very start of the member's employment, when service $n = 0$, so both ALs are 0, and at the very last retirement date, when the FAS is the same under both PUC and TUC.

    (ii)   The TUC NC will be <u>greater</u> than the PUC NC.

- The TUC normal cost each year starts below the PUC, but around 2/3 through the max employment period, the lines cross. In Barry's case, as he is near to retirement, the TUC NC will be greater than the PUC NC.

Comments: Try something! In that case, half of the points went for answering "bigger" or "smaller" correctly. When it comes to giving reasons, be careful! Your reasoning needs to be consistent. No contradicting yourself and no word dump! Many candidates did not appear to know the difference between PUC (projected unit credit) and TUC (traditional unit credit).

(d)

$$(AL_{2022} + NC_{2022})(1.051) = (167{,}151.26 + 13{,}269.11)(1.051) = 189{,}621.81$$

$$13 \times 0.016 \times 72{,}100 \times (1 - 12 \times 0.004) \times 13.3735$$
$$= 14{,}276.95 \times 13.3735 = 190{,}932.84$$

$$189{,}621.81 - 190{,}932.84 = -1311.03$$

The plan made a loss of 1311 from the early retirement.

OR (gain/loss analysis)

Interest component $= (AL_{2022} + NC_{2022})(1.051) - (AL_{2022} + NC_{2022})(1.05)$
$$= (167{,}151.26 + 13{,}269.11) \times 0.001 = 180.42$$

No ret at 63.5 component $= \frac{r_{63}}{l_{63}} EPVR_{63.5}(1.05)^{0.5} - \frac{r_{63}}{l_{63}} AL_{2023}$
$$= \frac{4{,}515.2}{47{,}579.3}[178{,}206.18(1.05)^{0.5} - 191{,}309] = -825.81$$

No other exits between 63 and 64 component $= 0 - \frac{i_{63} + d_{63}}{l_{63}} AL_{2023}$
$$= -\frac{45{,}2 + 213{,}9}{47{,}579.3}(191{,}309) = -1041.80$$

Ret at 64 component $= AL_{2023} - EPVR_{64}$
$$= 191{,}309 - 13 \times 0.016 \times 72{,}100 \times (1 - 12 \times 0.004) \times 13.3735$$
$$= 376.16$$

Total $= 180.42 - 825.81 - 1041.80 + 376.16 = -1311.03$

The plan made a loss of 1311 from the early retirement.

Comments: Very few candidates even attempted this part in a meaningful way. In that case, the key notion in assessing gain or loss is figuring out the difference between the money available versus the amount needed. If you have more than available, it is a gain; otherwise, it is a loss. It could have been more explicit that the funding level at BOY was 100%. From there, you need to find out funds available at the time of retirement versus what is needed to fund that retirement.

Question 6 Model Solution

(a) (i) $_1q_{[95]} = 1 - \dfrac{l_{96}}{l_{[95]}} = 0.45$

(ii) $_1q_{95} = 1 - \dfrac{l_{96}}{l_{95}} = 0.2143$

(iii) Typical select tables used for life insurance pricing assume lighter mortality during the select period, as the underwriting process selects lives that are healthier than average at inception. In this table, the mortality during the select period is heavier than ultimate mortality.

Comments: Candidates generally did well on part A. We awarded partial credit for candidates calculating the probability of survival, not death. On part iii we were looking for commentary on the $q_x s$ themselves, and not necessarily the structure of the table (since select and ultimate tables come in many varieties).

(b)

(i)

t	$_{t-1}V$	Prem	Annuity	Exp	Int	E_tV	Pr_t
0				300			−300
1	0	110,000	50,000	100	4,193	57,750	6,343
2	105,000	--	50,000	100	3,843	56,000	2,743

$$E_1V = p_{[95]} \times 105,000 = 57,750; \quad E_2V = p_{96} \times 80,000 = 56,000$$

(ii)

$\Pi_0 = Pr_0 = -300; \quad \Pi_1 = Pr_1 = 6,343; \quad \Pi_2 = p_{[95]}Pr_2 = 1,509;$
$\Pi_3 = {_2p_{[95]}}Pr_3 = 130; \quad \Pi_4 = {_3p_{[95]}}Pr_4 = 82$

$$where \ p_{[95]} = \frac{550}{1000}, \ _2p_{[95]} = \frac{385}{1000}, \ and \ _3p_{[95]} = \frac{195}{1000}$$

$$NPV(1) = NPV(0) + \Pi_1 \, v_{12\%} = 5{,}363;$$
$$NPV(2) = NPV(1) + \Pi_2 \, v_{12\%}^2 = 6{,}566;$$
$$NPV(3) = NPV(2) + \Pi_3 \, v_{12\%}^3 = 6{,}659;$$
$$NPV(4) = NPV(3) + \Pi_4 \, v_{12\%}^4 = 6{,}711$$

(iii)   The profit margin is 6,711/110,000=0.0610

Comments: Candidates also generally did well here. The most common errors were calculating the expected reserve and profit incorrectly, which a review of the material in the text would sort out easily enough. Checking their own work on part iii (ie, did the profit margin they calculated match the one asked for in the question) would have awarded more points as well.

(c) We now have

t	$_{t-1}V$	Prem	Annuity	Exp	Int	EtV	Pr$_t$
1	0	P	--	--	0.07P	30,195	1.07P-30,195
2	54,900	--	--	--	3,843	56,000	2,743

All the Π_t values are the same as in (a), except for the first year:

$$\Pi_1 = Pr_1 = 1.07P - 30{,}195$$

$$NPV = 0.061P = 6711 - 6343v_{12\%} + (1.07P - 30{,}195)v_{12\%}$$

$$\Rightarrow P = 28{,}973$$

Comments: Many candidates did not attempt this question. For those who did, some incorrectly attempted to use recursion to calculate a new premium based on then new reserve, which doesn't work in part because of the constraint to maintain the same profit margin. If candidates did well on part B, they generally also did well on C.

(d) Disadvantages:

Under the deferred annuity, the policyholder would pay 78,973 (annuity plus premium) in the first year, 50,000 in the second year, conditional on survival, and nothing thereafter. The total cost for a survivor, is 128,973. Under the annuity-due, the total cost is 110,000. So, for a 1-year survivor, even allowing for interest, the deferred annuity is more expensive.

If death occurs in the first two years, the annuitant and/or beneficiaries will receive nothing, losing the single premium.

Advantages:

An advantage of the deferred annuity is that it is less costly for lives who die in the first year, whose outlay is 78,973 compared with 110,000 for the annuity-due, thereby leaving more available for bequest under the SPDA, in the event of early death.

The single premium of a deferred annuity is smaller/cheaper than for an annuity due.

Comments: Full marks were given for any reasonable answer. Some candidates responded from the insurance company's perspective and not the insured, and these responses were given no credit.